Patterns #7

Recursive vs. explicit formulas

Definition

A **recursive formula** for a sequence $a_1, a_2, a_3, a_4, a_5, \ldots$ is a formula for computing a_n using previous terms of the sequence as well as the variable n. Examples of recursive formulas are $a_n = 2a_{n-1} + 5a_{n-2}$ or $a_n = 3a_{n-1} + n$.

An **explicit formula** for a_n only uses the variable n. An example of an explicit formula is $a_n = n + 2^n$.

Exercise

Each sequence below is described with a recursive formula. Write out the first six terms of each sequence.

1. $a_n = 2a_{n-1} + 1$ with $a_1 = 3$

2. $a_n = a_{n-1} + n$ with $a_1 = 7$

3. $a_n = a_{n-1} + a_{n-2}$ with $a_1 = 4$ and $a_2 = 7$

4. $a_n = a_{n-1} \cdot a_{n-2}$ with $a_1 = 2$ and $a_2 = 3$

Exercise

Suppose a sequence has a recursive formula $a_n = a_{n-1} + 7$ with $a_1 = 4$. Write out the first 5 terms of the sequence. Do you recognize it as being arithmetic or geometric? Write an *explicit* formula for the sequence.

Exercise

Suppose a sequence has a recursive formula $a_n = 5a_{n-1} - 6a_{n-2}$ with $a_1 = 2$ and $a_2 = 5$. Write out the first 5 terms of the sequence. Confirm that an *explicit* formula for the sequence is given by $a_n = 2^{n-1} + 3^{n-1}$.

Exercise

Consider the sequence $2, 6, 12, 20, 30, \ldots$.

1. What is the pattern? Use it to fill in the table and find a *recursive* formula for the sequence.

Term	**Work to compute the term *from previous terms***	**Value of the term**
a_1	N/A	2
a_2	$a_2 = a_1 + 4$	6
a_3	$a_3 =$	12
a_4	$a_4 =$	20
a_5	$a_5 =$	30
$\vdots$	$\vdots$	$\vdots$
n	$a_n =$	N/A

2. Now look at the number of blocks in the sequence of figures below. What do you notice? Use this to find an *explicit* formula for the sequence above.

Figure 1 Figure 2 Figure 3 Figure 4