01 – Row Echelon Form

Definition: Row Echelon Forms

A matrix A is in **row echelon form (REF)** if

1. all nonzero rows lie above any rows of all zeros;

2. the leading entry (from the left) of each nonzero row is strictly to the right of the leading entry of the row above it.

A matrix A is in **reduced row echelon form (RREF)** if, in addition to **1.** and **2.**, it also satisfies

3. the leading entry (from the left) of each nonzero row is a 1 (called the **leading one**);

4. each leading one is the only nonzero entry in its column.

1. Determine if each of the following are in REF or RREF.

(a) $\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(e) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 7 & 6 \\ 2 & 3 & 4 \end{bmatrix}$

(b) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 5 \\ 0 & 6 & 7 \end{bmatrix}$

(d) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(f) $\begin{bmatrix} 3 & 0 & 1 & 6 \\ 0 & 2 & 4 & 3 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

Definition: Elementary Row Operations

An **elementary row operation** on a matrix is any of the following.

Replacement: add to one row any multiple of another row $(cr_i + r_j \to r_j)$

Interchange: interchange two rows $(r_i \leftrightarrow r_j)$

Scale: multiply a row by a nonzero scalar $(cr_i \to r_i)$

2. Look back at the matrices in the previous example.

(a) For each matrix that was not in REF, find a sequence of elementary row operations that could be used to transform it into REF.

(b) For each matrix that was already in REF, find a sequence of elementary row operations that could be used to transform it into RREF.