

02 – Row Reduction

Strategy: Row Reduction

To transform a matrix to REF or RREF, use the following algorithm.

1. Find the leftmost nonzero column—this is called the **pivot column**.
2. Choose a nonzero entry in the pivot column—this will be called the **pivot**. If necessary, use INTERCHANGE operations to make sure the pivot is in the top row.
3. Use REPLACEMENT operations to create zeros *below* the pivot.
4. Cover up (or ignore) the row containing the pivot, and repeat steps 1–3 on the smaller matrix below. Continue repeating until there are no more nonzero rows to modify.

At this point, the matrix is in REF. To transform to RREF, continue with the process below.

5. Beginning with the rightmost pivot, working up and to the left, use REPLACEMENT operations to create zeros *above* each pivot. If a pivot is not 1, use a SCALING operation to make it 1.

At this point, the matrix is in RREF.

1. Row reduce the following matrix to RREF, and determine if the corresponding linear system is consistent or not.

$$A = \begin{bmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{bmatrix}$$

2. Solve the following linear system by reducing the corresponding augmented matrix to RREF.

$$x_2 + x_3 = 2$$

$$-3x_1 + 2x_2 = 4$$

$$x_1 + x_3 = 1$$

3. Solve the system given in augmented matrix form as $\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$.

Definition: Basic and Free Variables

If A is the augmented matrix of a linear system, then

- the variables corresponding to pivot columns are called **basic variables**, and
- the variables corresponding to columns with NO pivot are called **free variables**.

Theorem: Existence and Uniqueness Theorem

- A linear system is inconsistent if and only if the RREF has a row of the form $[0 \ 0 \ \cdots \ 0 \ b]$ with $b \neq 0$.
- If a linear system is consistent, then it has infinitely-many solutions if and only if there are free variables.