13 – Dimension

Theorem

If H is a nonzero subspace of $\mathbb{R}^n$, then every basis for H consists of the same number of vectors.

Definition: Dimension

If H is a nonzero subspace of $\mathbb{R}^n$, then the number of vectors in a basis for H is called the **dimension**, denoted $\dim H$. The dimension of the zero subspace $\{0\}$ is defined to be 0.

1. Determine the dimension of each of the following.

(a) $\mathbb{R}^3$

(b) $\mathbb{R}^n$

(c) $\text{Span}\left\{\begin{bmatrix}-1\\-2\\1\end{bmatrix}, \begin{bmatrix}2\\4\\-2\end{bmatrix}, \begin{bmatrix}3\\9\\-3\end{bmatrix}\right\}$

(d) $\text{Span}\left\{\begin{bmatrix}0\\1\\-2\end{bmatrix}, \begin{bmatrix}5\\-7\\4\end{bmatrix}, \begin{bmatrix}6\\3\\5\end{bmatrix}\right\}$

Definition: Rank & Nullity

Let A be a matrix.

- The **rank** of A is the dimension of $\text{Col}(A)$.
- The **nullity** of A is the dimension of $\text{Nul}(A)$.

2. Let $A = \begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & -1 & 5 \\ 2 & -6 & 4 & -3 \\ 4 & -12 & 2 & 7 \end{bmatrix}$

(a) Determine the rank of A.

(b) Determine the nullity of A.

Theorem

Let A be an $m \times n$ matrix. Let p denote the number of pivot positions in A.

1. $\text{rank}\, A =$ ______________

2. $\text{nullity}\, A =$ ______________

3. Rank-Nullity Theorem: $\text{rank}\, A + \text{nullity}\, A =$ ______________