16 – Diagonalization Theorem

Definition

A matrix A is **diagonalizable** if $A = PDP^{-1}$ (or equivalently $D = P^{-1}AP$) for some *diagonal* matrix D and some invertible matrix P.

Theorem: Diagonalization Theorem

Let A be an $n \times n$ matrix.

1. A is diagonalizable if and only if A has n linearly independent eigenvectors.

2. If A is diagonalizable and $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly independent eigenvectors for A with corresponding eigenvalues $\lambda_1, \ldots, \lambda_n$, then $A = PDP^{-1}$ for

$$P = \begin{bmatrix} \mathbf{v}_1 & \cdots & \mathbf{v}_n \end{bmatrix} \qquad \text{and} \qquad D = \begin{bmatrix} \lambda_1 & & \text{\huge 0} \\ & \ddots & \\ \text{\huge 0} & & \lambda_n \end{bmatrix}$$

1. Diagonalize the following, if possible.

(a) $B = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Theorem

Let A be an $n \times n$ matrix. If A has n different eigenvalues, then A is diagonalizable.

2. Explain why each of the following are diagonalizable.

(a) $A = \begin{bmatrix} 5 & 0 & 0 \\ 2 & 0 & 0 \\ 7 & 8 & \pi \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$