03 – Linear Combinations and Span

Definition: Linear Combination

Let $\mathbf{v}_1, \dots, \mathbf{v}_k$ be vectors in $\mathbb{R}^n$. A vector $\mathbf{w}$ is called a **linear combination** of $\mathbf{v}_1, \dots, \mathbf{v}_k$ if it can be written in the form

$$\mathbf{w} = c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k$$

where $c_1, \dots, c_k$ are scalars in $\mathbb{R}$.

1. Let $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$

(a) Write down three different linear combinations of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$.

(b) Is $\begin{bmatrix} -3 \\ 6 \\ -6 \end{bmatrix}$ a linear combinations of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$?

(c) Is $\begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$ a linear combinations of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$?

2. Suppose you want to determine if a vector $\mathbf{b}$ is a linear combination of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_k$. Describe how to solve this type of problem using a linear system.

Definition: Span

Let $\mathbf{v}_1, \ldots, \mathbf{v}_k$ be vectors in $\mathbb{R}^n$. The set of *all* possible linear combinations of $\mathbf{v}_1, \ldots, \mathbf{v}_k$ is denoted $\mathrm{Span}\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$. It is called the **subset spanned** by $\mathbf{v}_1, \ldots, \mathbf{v}_k$ or simply the **span** of $\mathbf{v}_1, \ldots, \mathbf{v}_k$.

3. Let $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ be as in Exercise **1.** Is $\begin{bmatrix} -3 \\ 6 \\ -6 \end{bmatrix}$ in $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$? Is $\begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$ in $\mathrm{Span}\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$?

4. Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \mathbf{a}_2 = \begin{bmatrix} 0 \\ 3 \\ 6 \end{bmatrix}, \mathbf{a}_3 = \begin{bmatrix} -4 \\ -2 \\ 3 \end{bmatrix}$. Determine if $\begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$ is in $\mathrm{Span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$.