MATH 102—OUTLINE FOR EXAM 1

Section 1 through beginning of Section 4

Definitions and Theorems

One thing I hope you all take away from this course is a fluency in the language of number theory. To that end, you are expected to **be able to write** the definitions of the following terms and the statements of the following theorems on the exam.

- definition of what it means that a *divides* b, i.e. $a \mid b$
- definition of the *greatest common divisor* of two integers a and b
- definition of a *prime* number
- definition of what it means that *a is congruent to b modulo m*, i.e. $a \equiv b \pmod{m}$
- statement of the *Division Algorithm*
- statement of *Theorem 4 of Section 1* (the "GCD Theorem")
- statement of the *Fundamental Theorem of Arithmetic* (also known as the *Unique Factorization Theorem*)

Problems to Practice

1. Computing the greatest common divisor of two integers using the Euclidean Algorithm (Section 1)
2. Finding primes and determining if a number is prime (Section 2)
   - Lemma 4 of Section 2 is very useful
3. Finding and using the prime-power decomposition of a number (Section 2)
4. Solving linear Diophantine equations (Section 3)
   - be able to write out all *integer* solutions (if any) to an equation of the form $ax + by = c$
     - remember, you may have to reduce it first to make sure you get *all* solutions
   - know how to quickly check if $ax + by = c$ has a solution using Lemma 2 of Section 3
   - be able to work with systems of equations with more than two variable
   - be able to solve these in the context of a word problem too
5. Working with basic congruences (beginning of Section 4)
   - be able to check if two integers are congruent modulo m
   - be able to find the least residue of an integer modulo m
6. Practice some proofs too!
   - Make sure you can reprove all proofs from the homework. I may or may not ask you to prove the exact same thing, but I will probably choose something similar.

How to study

1. Memorize the definitions and theorems listed above and practice writing them out
2. Review core topics—make sure to have a working understanding of all definitions and theorems
3. Work problems all of the way through—focus on ones similar to those from Homeworks 1–4 and the Warm-Ups
4. Practice proofs—focus on ones similar to those from Homeworks 1–4 and the Warm-Ups
5. Come talk with me if you have any questions