

MATH 102—OUTLINE FOR EXAM 1

Sections covered: Section 1 through beginning of Section 4

Definitions and Theorems

On the exam, I will ask you to carefully write the statement of 2–3 definitions and 1 theorem. I will choose the definitions and theorems from the list below.

- definition of *a divides b*, i.e. $a \mid b$
- definition of the *greatest common divisor* of two integers a and b
- definition of a *prime* number
- definition of *a is congruent to b modulo m*, i.e. $a \equiv b \pmod{m}$
- statement of the *Division Algorithm*
- statement of *Theorem 4 of Section 1* (the “GCD Theorem”)
- statement of the *Fundamental Theorem of Arithmetic* (also known as the *Unique Factorization Theorem*)

Problems to Practice

1. Computing the greatest common divisor of two integers using the Euclidean Algorithm (Section 1)
2. Finding primes and determining if a number is prime (Section 2)
 - Lemma 4 of Section 2 is very useful
3. Finding and using the prime-power decomposition of a number (Section 2)
4. Solving linear Diophantine equations (Section 3)
 - be able to write out all *integer* solutions (if any) to an equation of the form $ax + by = c$
 - remember, you may have to reduce it first to make sure you get *all* solutions
 - know how to quickly check if $ax + by = c$ has a solution using Lemma 2 of Section 3
 - be able to work with systems of equations with more than two variables
 - be able to solve these in the context of a word problem too
5. Working with basic congruences (beginning of Section 4)
 - be able to check if two integers are congruent modulo m
 - be able to find the least residue of an integer modulo m
6. Practice proofs too!
 - I will ask you to do 1–2 proofs on the exam. They will be similar to something you have done in homework or we have done in class.
 - Make sure you can reprove all proofs from the homework.
 - Look over the proofs we did in class, but some of them are harder than what I will ask for on the exam.

How to study

1. Memorize the definitions and theorems listed above and practice writing them out.
2. Review core topics—make sure to have a working understanding of all definitions and theorems.
3. Work problems all of the way through—focus on ones similar to those from Homeworks 1–4 and the Warm-Ups.
4. Practice proofs—focus on ones similar to those from Homeworks 1–4 and the Warm-Ups.
5. Please talk with me or email me if you have any questions!