Math 108—Outline for Final Exam

Sections: 1.1–1.7, 2.1–2.5, 3.1–3.4, 4.1–4.4, Intro. to Cardinality

1 Language

One thing that I hope you all take away from this course is a fluency in the language of logic and set theory. To that end, you are expected to be able to recite the definitions for the following terms as well as the statements of the following theorems. Keep in mind that you are also expected to have a "working understanding" of **everything** that we have covered.

Terms

- a (mathematical) *proposition*
- the *converse* and *contrapositive* of an implication $P \implies Q$
- what it means to say that a set A is a *subset* of a set B ($A \subseteq B$)
- what it means to say that two sets A and B are *disjoint*
- the *complement* and the *power set* of a set A
- the *union*, *intersection*, *difference*, and *product* (or *Cartesian product*) of sets A and B
- the *union* and *intersection* over a family of sets $\mathcal{A}$
- a *relation* from A to B
- the *inverse* of a relation from A to B
- the *composition* (denoted $S \circ R$) of a relation R from A to B with a relation S from B to C
- the *reflexive*, *symmetric*, and *transitive* properties for relations on a set
- an *equivalence relation* on a set
- the *equivalence class of x modulo R* when R is an equivalence relation
- a *partition* of a set
- the *restriction* of a function $f : A \to B$ to a subset D of A.
- *injection*, *surjection*, and *bijection*
- what it means to say that two sets A and B are *equivalent* ($A \approx B$)

Theorems

- **De Morgan's Laws**: if P and Q are propositions, then
$$\sim (P \wedge Q) \equiv (\sim P) \vee (\sim Q) \quad \text{and} \quad \sim (P \vee Q) \equiv (\sim P) \wedge (\sim Q).$$
- **Principle of Mathematical Induction (PMI)**: if $S \subseteq \mathbb{N}$ such that
  (i) $1 \in S$, and
  (ii) for all $n \geq 1$, $n \in S$ implies $n + 1 \in S$,
  then $n \in S$ for all $n \geq 1$.
- **Theorem 3.2.2 (as presented in class)**: if R is an equivalence relation on a nonempty set A, then for all $x, y \in A$
  (1) $x \, R \, y$ if and only if $\bar{x} = \bar{y}$, and
  (2) either $\bar{x} = \bar{y}$ or $\bar{x} \cap \bar{y} = \emptyset$.
- **Theorem 4.4.2 (as presented in class)** If f is a function from A to B, then f^{-1} is a function from B to A if and only if f is a bijection.

2 Problems to Practice

From Exam 1

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1. Translating to and from symbolic logic

2. Proving two propositional forms are equivalent with a truth table

3. Using set-builder notation

4. "True or False" problems

  (a) Determining truth values of propositions (especially involving quantifiers)
  (b) Determining if an element is contained in a given set (or power set)
  (c) Determining if a set is contained in a given set

5. "Example or no such example exists" problems

  (a) Examples of elements and/or sets meeting certain set-theoretic criteria
  (b) Practice "prove or disprove" problems

From Exam 2

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6. Working with Cartesian products

7. Working with unions and intersections over families of sets

8. Working with relations

9. Working with equivalence relations (for example, determining the equivalence class of an element)

10. Checking if a relation is reflexive, symmetric, and transitive

11. Checking if a collection of subsets of A is a partition of A

12. Modular arithmetic (in $\mathbb{Z}_m$)

13. "Example or no such example exists" problems

  (a) Examples of elements, sets, and/or relations meeting certain criteria
  (b) Practice "prove or disprove" problems

New Material

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14. Proving a relation is a function

15. Determining if a function is an injection or surjection (or both)

16. Determining if a function has an inverse

3 Proofs

From Exam 1

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1. Make sure you could reprove all proofs from class, groupwork, homework, and writing assignments

2. Practice choosing which proof technique to use (I probably won't tell you to use contraposition or contradiction)

3. Practice using cases (the method of exhaustion)

4. Practice "prove or disprove" problems (these are easy to make up on your own)

5. Practice how to prove that a set is contained in another set and how to prove that two sets are equal

From Exam 2

6. Practice using PMI

7. Practice proving (or disproving) the reflexive, symmetric, and transitive properties for a relation

8. Practice “prove or disprove” problems (these are easy to make up on your own)

9. Re-practice the “basics,” for example:

- how to prove implications
- how to prove that a set is contained in another set or that two sets are equal

I know that you’ve already practiced these, but they take a while to become natural.

10. Make sure you could reprove all proofs from class, group work, homework, and writing assignments

New Material

11. Practice how to prove or disprove that a function is an injection or surjection (or both)

12. Proving or disproving $A \approx B$