Math 108—Homework 02

Due: Tuesday February 7

Name ______________________

Directions: please print this page, and put your solutions in the space provided.

1. Let $L(x, y)$ be the statement "x loves y." where the universe for both x and y is the set of all people in the world. Use quantifiers to express each of the following statements.

(a) Everybody loves somebody.

(b) Nobody loves everybody.

(c) There is somebody whom Lydia does not love.

(d) There is someone who loves no one besides themself.

2. Let $P(x)$ be the statement "x has an Internet connection" and $C(x, y)$ be the statement "x and y have chatted over the Internet," where the universe for both x and y is the set of all students in this class. Use quantifiers to express each of the following statements.

(a) No one in the class has chatted with Bob.

(b) Not everyone in the class has an Internet connection.

(c) Someone in the class has an Internet connection but has not chatted with anyone else in the class.

(d) There is a student in the class who has chatted on the internet with everyone in the class who has a connection.

3. Write down **the negation of each of the following statements** in such a way that *negation symbols only appear next to the predicates* P, Q, or R.

(a) $\exists x(\sim P(x) \wedge Q(x))$

(b) $\forall x(P(x) \implies (\exists y(Q(y)) \wedge R(x, y))))$

4. Determine (with justification) the truth value of each of the following statements if the universe for all variables is $\mathbb{Z}$, the set of all integers.

(a) $\exists n\ \forall m\ (n < m^2)$

(b) $\exists n\ \forall m\ (nm = m)$

(c) $\exists n\ \exists m\ (n^2 + m^2 = 5)$

(d) $\exists n\ \forall m\ (n + m = 0)$

(e) $\exists n\ \exists m\ [(n + m = 4) \wedge (n - m = 1)]$

(f) $\forall n\ \forall m\ \exists p\ \left(p = \frac{m+n}{2}\right)$

5. Determine (with justification) the truth value of each of the following statements if the universe for all variables is $\mathbb{R}$, the set of all real numbers.

(a) $\exists x\ \forall y\ (xy = 0)$

(b) $\exists x\ \exists y\ (x + y \neq y + x)$

(c) $\exists x\ \forall y \neq 0\ (xy = 1)$

(d) $\forall x\ \exists y\ (x + y = 1)$