Math 108—Homework 05

Due: Tuesday February 28

Name ______________________

Directions: please print this page, and put your solutions in the space provided.

1. Let $X = \{\pi, e, \{1, 2\}\}$, and let $\mathcal{P}(X)$ denote the power set of X. Determine if each is **True** or **False**.
You do not need to justify your answers, but make sure to double check them.

(a) $\pi \in X$

(b) $\{\pi\} \in X$

(c) $\{\pi\} \subseteq X$

(d) $2 \in X$

(e) $\{1, 2\} \subseteq X$

(f) $\{\pi, e\} \in \mathcal{P}(X)$

(g) X contains 4 elements.

(h) $\mathcal{P}(X)$ contains 8 elements

2. Let $A = \{1, 2\}$. Give an example (if one exists) of sets B and C such that the following are true. If no example exists, simply write "not possible."

(a) $A \subseteq B$, $A \subseteq C$, and $B \nsubseteq C$

(b) $A \subseteq B$, $A \nsubseteq C$, and $B \subseteq C$

(c) $A \subseteq B$, $B \subseteq C$, and $C \subseteq A$

(d) $A \in B$, $A \nsubseteq B$, $A \subseteq C$, $B \subseteq C$

3. Let $\mathcal{P}(\mathbb{Z})$ denote the power set of $\mathbb{Z}$. Give an example of each of the following:

(a) an $A \in \mathcal{P}(\mathbb{Z})$ where A contains 3 elements.

(b) an $A \in \mathcal{P}(\mathbb{Z})$ where A contains infinitely many elements.

(c) an $A \subseteq \mathcal{P}(\mathbb{Z})$ where A contains 3 elements.

(d) an $A \subseteq \mathcal{P}(\mathbb{Z})$ where A contains infinitely many elements.

4. For each set, find a universe U and a formula $P(x)$ so that the given set is equal to $\{x \in U : P(x)\}$.

I did the first one for you.

(a) $A = \{\ldots, -9, -4, 1, 6, 11, \ldots\}$

Possible answer: $A = \{x \in \mathbb{Z} : \exists m \in \mathbb{Z}(x = 1 + 5m)\}$

(b) $B = \{4, 9, 16, 25, 36, \ldots\}$

(c) $C = \{\text{all rational numbers that are not integers}\}$

(d) $D = \{\pi, e, \sqrt{2}\}$

5. For $a \in \mathbb{Z}$, define $a + 5\mathbb{Z} = \{x \in \mathbb{Z} : \exists m \in \mathbb{Z}(x = a + 5m)\}$.

- Notice that $0 + 5\mathbb{Z}$ is the same as $5\mathbb{Z}$, as we defined it in class.
- For another example, $2 + 5\mathbb{Z} = \{\ldots, -8, -3, 2, 7, 12, \ldots\}$.

Prove: for all $a, b \in \mathbb{Z}$, if $(a - b) \in 5\mathbb{Z}$, then $a + 5\mathbb{Z} = b + 5\mathbb{Z}$.