MATH 108—HOMEWORK 06

Due: Tuesday March 04

NAME _____

Directions: please print this page, and put your solutions in the space provided.

1. The universe is $\mathbb{Z}$. Let $2\mathbb{Z} = \{$all even integers$\}$, $\mathbb{Z}^+ = \{$all positive integers$\}$ and $\mathbb{Z}^- = \{$all negative integers$\}$. *Note that 0 is not positive and not negative.* Give an example (if one exists) of each of the following; if no example exists, simply write "not possible."

(a) an $x \in (2\mathbb{Z} - \mathbb{Z}^+)$

(c) an $x \in ((\mathbb{Z}^- \cap \mathbb{Z}^+)^c \cap \{-1, 0, 1\})$

(b) an $x \in (\mathbb{Z}^- - 2\mathbb{Z})$

(d) an $x \in (2\mathbb{Z} - (2\mathbb{Z} - \mathbb{Z}^-))$

(e) an $x \in ((2\mathbb{Z})^c \cap \{a \in \mathbb{Z} : \exists m \in \mathbb{Z}(a = 2^m)\})$

(f) an $x \in (\mathbb{Z}^- \cap \{a \in \mathbb{Z} : [\forall m \in \mathbb{Z}(am = a)] \vee [\forall m \in \mathbb{Z}(am = -a)]\})$

2. Let $A = \{1, 2, 3\}$. Give an example (if one exists) of sets B and C such that the following are true. If no example exists, simply write "not possible."

(a) $C \subseteq A \cup B$, $A \cap B \nsubseteq C$

(c) $A \cap B \subseteq C$, $A \cap C \subseteq B$, $A = B \cup C$

(b) $B - A = C$, A is disjoint from B

(d) $A \subseteq B$, $C - A = \emptyset$, $C - B \neq \emptyset$

3. Prove or disprove: for all sets A, B, and C, if $(A \cup C) \subseteq (B \cup C)$, then $A \subseteq B$.

4. **Prove or disprove:** for all sets A, B, and C, $(A - B) - C = (A - C) - (B - C)$.

5. **Prove:** for all $a, b \in \mathbb{Z}$, the sets $a + 5\mathbb{Z}$ and $b + 5\mathbb{Z}$ are either equal or disjoint.

   Hint: if $a + 5\mathbb{Z}$ and $b + 5\mathbb{Z}$ are not disjoint, then there exists some $x \in (a + 5\mathbb{Z}) \cap (b + 5\mathbb{Z})$. Now, use this to prove that the sets are equal. (Earlier results may be very helpful.)