# Math 108—Homework 07

*Due: Thursday March 16*

Name ______________________________

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**Directions: please print this page, and put your solutions in the space provided.**

**1.** Let $A = \{1, 2, 3\}$. Give an example (if one exists) of sets $B$, $C$, and $D$ such that the following are true. If no example exists, simply write "not possible."

**(a)** $D \subseteq A \times A$, $|D| = 4$

**(b)** $D \subseteq A \times A$, $D \subseteq A$

**(c)** $C \times C \subseteq A \times B$, $|A \times B| = 15$

**(d)** $C \times C \subseteq A \times B$, $|C \times C| = 15$

**2.** Let $\mathcal{B} = \{B_n : n \in \mathbb{Z}\}$ where $B_n = (n, n+1)$; $B_n$ is an interval of *real* numbers.

**(a)** Find $B_0 \cup B_1$.

**(b)** Find $\bigcup_{B \in \mathcal{B}} B$.

**(c)** Find $\bigcap_{B \in \mathcal{B}} B$.

**3.** Let $\mathcal{A} = \{A_n : n \in \mathbb{N}^+\}$ where $A_n = (-n, \frac{1}{n})$; $A_n$ an interval of *real* numbers.

**(a)** Find $A_1 \cap A_2$.

**(b)** Find $\bigcup_{A \in \mathcal{A}} A$.

**(c)** Find $\bigcap_{A \in \mathcal{A}} A$.

**4.** Let $\mathcal{C} = \{k\mathbb{Z} : k \in \mathbb{Z} \text{ with } k \geq 2\}$ where $k\mathbb{Z} = \{x \in \mathbb{Z} : \exists m \in \mathbb{Z}(x = km)\}$.

**(a)** Find $2\mathbb{Z} \cap 3\mathbb{Z}$.

**(b)** Find $\bigcup_{C \in \mathcal{C}} C$.

**(c)** Find $\bigcap_{C \in \mathcal{C}} C$.

**5.** Let $X = \{1, 2, 3, 4, \ldots, 20\}$. Give an example of each of the following:

**(a)** a family $\mathcal{A}$ of subsets of $X$ such that $\bigcup_{A \in \mathcal{A}} A = X$ and $\bigcap_{A \in \mathcal{A}} A = \{1\}$.

**(b)** a family $\mathcal{B}$ of consisting of **four pairwise disjoint** subsets of $X$ such that $\bigcup_{B \in \mathcal{B}} B = X$.

**6.** Give an example of each of the following:

**(a)** a family $\mathcal{A}$ of **open intervals** in $\mathbb{R}$ such that $\bigcup_{A \in \mathcal{A}} A = \mathbb{R}$ and $\bigcap_{A \in \mathcal{A}} A = [-2, 2]$.

**(b)** a family $\mathcal{B}$ of consisting of **infinitely many pairwise disjoint intervals** of $\mathbb{R}$ such that $\bigcup_{B \in \mathcal{B}} B = \mathbb{R}$.

**7. Prove:** $B \cap \bigcup_{A \in \mathcal{A}} A = \bigcup_{A \in \mathcal{A}} (B \cap A)$.

*The right-hand side is a little confusing. This is what it means:* $\bigcup_{A \in \mathcal{A}} (B \cap A) = \{x : x \in B \cap A \text{ for some } A \in \mathcal{A}\}$.