

# MATH 108—HOMEWORK 08

Due: Thursday April 6

NAME \_\_\_\_\_

**Directions: please print this page, and put your solutions in the space provided.**

1. Consider the relation  $R$  on  $\{-1, 0, 1, 2, 3, 6, 9\}$  defined by  $x R y \iff x^2 = y$ .

(a) Write  $R$  as a list of ordered pairs.

(b) Write  $R^{-1}$  as a list of ordered pairs.

(c) Draw the graph of  $R$ .

(d) Draw the digraph associated to  $R$ .

2. Consider the relation  $R$  on  $\mathbb{Z}^+$  defined by  $x R y \iff$  “ $x$  divides  $y$ .”

(a) Find the set of all  $y$  such that  $2 R y$ .

(c) Find the set of all  $x$  such that  $x R 30$ .

(b) Find the set of all  $y$  such that  $5 R y$ .

(d) Find the set of all  $x$  such that  $x R 17$ .

3. Consider the following relations on  $\mathbb{Z}$ :

•  $R = \{(-1, 1), (2, 0), (3, 3), (-1, 0), (3, -1)\}$

•  $S = \{(0, 0), (3, -1), (0, -7), (1, 1)\}$

(a) Find  $S \circ R$ .

(c) Find  $R \circ R$ .

(b) Find  $R \circ S$ .

(d) Find  $R^{-1} \circ R$ .

4. Prove that if  $A$  is a set and  $R$  is a relation on  $A$ , then the relation  $S$  defined by  $S = R \cup R^{-1}$  is symmetric.

5. Let  $f_n$  be the  $n^{\text{th}}$  Fibonacci number. Recall that the Fibonacci numbers are defined by

$$f_1 = 1, f_2 = 1, \text{ and } f_n = f_{n-1} + f_{n-2} \text{ for } n \geq 2.$$

Prove that  $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$  for all  $n \geq 1$ .