Math 108—Homework 09

Due: Thursday April 13

Name _____

Directions: please print this page, and put your solutions in the space provided.

1. Determine if each of the following relations are reflexive, symmetric, or transitive.

- If you believe the relation has a property, you can just say so, without proof.
- If you believe the relation does not have a property, give an example showing that the property fails.

(a) The relation R on $\mathbb{R}$ defined by $x\,R\,y \iff |\,x-y\,| < 1$.

(b) The relation R on $\mathbb{R}$ defined by $x\,R\,y \iff \exists z \in \mathbb{R}$ such that $x - y = z^2$.

(c) The relation R on $\mathbb{Z}$ defined by $x\,R\,y \iff x^2 + y^2$ is even.

2. Determine if P is a partition of A.

- If you believe that P is a partition, you can just say so, without proof.
- If you believe that P is not a partition, explain why not.

(a) $A = \mathbb{R}^+$, $P = \{A_k : k \in \mathbb{Z}^+\}$ where A_k is the interval $A_k = \left(\frac{1}{k}, k\right)$.

(b) $A = \mathbb{R} \times \mathbb{R}$, $P = \{A_k : k \in \mathbb{R}\}$ where $A_k = \{(x, y) \in \mathbb{R} \times \mathbb{R} : y + x = k\}$.

3. Notice that every $x \in \mathbb{Z}^+$ can be written as $x = 2^k n$ where n is an odd number. This is saying that 2 appears k-times in the prime factorization of x. Define $\nu(x) = k$. (Just to check, $\nu(4) = 2$ and $\nu(60) = \nu(2^2 \cdot 3 \cdot 5) = 2$.) Now define a relation R on $\mathbb{Z}^+$ by $x \, R \, y \iff \nu(x) = \nu(y)$, and notice that R is an equivalent relation.

(a) List five different elements in the equivalence class of 4. (Note that we already saw $4 \, R \, 60$.)

(b) Find an element less than 10 in the equivalence class of 168.

4. Make the following computations in $\mathbb{Z}_7$. Write your answer as x with $0 \leq x \leq 6$. *Show your work!*

(a) $7 + 14 + 21 + 28 + 35 + 42 + 49 \pmod 7$

(b) $2^3 - 3^2 \pmod 7$

(c) $8^{12345} \pmod 7$

(d) $6^{12345} \pmod 7$

5. Solve the equation $2x = -1$ in $\mathbb{Z}_7$.