Math 108—Homework 10

Due: Tuesday April 25

Name _____

Directions: please print this page, and put your solutions in the space provided.

1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 1$

(a) Find the image of $\sqrt{2}$.

(b) Find a preimage of $\sqrt{2}$, if possible.

(c) Find a preimage of $-\sqrt{2}$, if possible.

(d) Find $\{x \in \mathbb{R} : f(x) = 2\}$ (all preimages of 2).

2. Let $\chi_{2\mathbb{Z}} : \mathbb{Z} \to \mathbb{Z}$ be the characteristic function of $2\mathbb{Z}$. *Recall that $2\mathbb{Z}$ is the set of even integers.*

(a) Find the image of 7532.

(b) Find a preimage of 7532, if possible.

(c) Find $\{x \in \mathbb{R} : \chi_{2\mathbb{Z}}(x) = 0\}$.

(d) Find $\{x \in \mathbb{R} : \chi_{2\mathbb{Z}}(x) = 1\}$.

3. Let f, g, and h, be defined as follows:

- $f : \mathbb{Z}_7 \to \mathbb{Z}_7$ be defined by $f(\bar{x}) = \bar{x}^2$
- $g : \mathbb{Z}_8 \to \mathbb{Z}_8$ be defined by $g(\bar{x}) = \bar{x}^2$
- $h : \mathbb{Z}_8 \to \mathbb{Z}_8$ be defined by $h(\bar{x}) = \overline{2x + 1}$

(a) Find $\mathrm{Rng}(f)$

(b) Find $\mathrm{Rng}(f \circ f)$

(c) Find $\mathrm{Rng}(h \circ g)$

(d) Find $\mathrm{Rng}(g \circ h)$

4. For every positive integer m, let $\tau : \mathbb{Z}_m \to \mathbb{Z}_m$ be defined by the rule $\tau(\bar{x}) = \overline{2x}$. (In words, τ is the function that multiples everything by 2.) **Prove** that if m is even, then τ^{-1} is *not* a function.