Math 108—Homework 11

Due: Thursday May 4

Name _____

Directions: please print this page, and put your solutions in the space provided.

1. Either provide an example of a function which satisfies the given description, or if no such example exists, briefly *explain* why not.

Hint: only three of them are "not possible."

(a) A function $f : \{a, b, c, d\} \to \mathbb{Z}_3$ such that f is onto $\mathbb{Z}_3$.

(b) A function $f : \{a, b, c, d\} \to \mathbb{Z}_3$ such that f is one-to-one.

(c) A function $f : (-\infty, 2] \to [-2, \infty)$ such that f is an injection.

(d) A function $f : \mathbb{R} \to \mathbb{R}^+$ such that f is one-to-one and onto $\mathbb{R}^+$.

(e) A function $f : \mathbb{Z}_5 \to \mathbb{Z}_5$ such that f is an injection but **not** a surjection.

(f) A function $f : \mathbb{Z} \to 5\mathbb{Z}$ such that f is onto $5\mathbb{Z}$.

(g) A function $f : \mathbb{Z} \to \mathbb{Z}_5$ such that f is one-to-one.

(h) A function $f : \mathbb{Z}^+ \to (\mathbb{Z}^+ - \{1\})$ such that f is an injection and a surjection.

(i) A function $f : \mathbb{Z} \to \mathbb{N}$ such that f is one-to-one and onto $\mathbb{N}$. (Please include 0 in $\mathbb{N}$, so $\mathbb{N} = \{0, 1, 2 \ldots\}$.)
Hint: it is possible! One option is to create a piecewise defined function of the following form.

$$f(n) = \begin{cases} 0 & \textit{if } n = 0 \\ ? & \textit{if } n \textit{ is positive} \\ ? & \textit{if } n \textit{ is negative} \end{cases}$$