Math 108—Homework 12

Due: Thursday May 11

Name _____

Directions: please print this page, and put your solutions in the space provided, or use the template here: https://www.sharelatex.com/project/590d251b8565aa37198d9ed1

1. Prove that $\mathbb{Z} \approx 2\mathbb{Z}$ by finding a bijection from $\mathbb{Z}$ to $2\mathbb{Z}$. (You need to **prove** that your function is a bijection.)

2. Prove that $[0, 1] \approx [0, 10]$ by finding a bijection from $[0, 1]$ to $[0, 10]$, where $[0, 1]$ and $[0, 10]$ are intervals of real numbers. (You need to **prove** that your function is a bijection.) *Hint: your function should be something you can graph—think of lines.*

3. Follow the steps below to prove that $f : \mathbb{Z}^+ \times \mathbb{Z}^+ \to \mathbb{Z}^+$ defined by $f(m, n) = 2^{m-1}(2n - 1)$ is a bijection. (This shows that $\mathbb{Z}^+ \times \mathbb{Z}^+ \approx \mathbb{Z}^+$.)

(a) Prove that f is a surjection.

Hint: begin with "Let $a \in \mathbb{Z}^+$. We will show that there exists $(m, n) \in \mathbb{Z}^+$ such that $f(m, n) = a$." I recommend considering two cases: a is even or a is odd. Experiment with actual numbers if you need.

(b) Prove that f is an injection.

Hint: begin with "Let $(m_1, n_1), (m_2, n_2) \in \mathbb{Z}^+ \times \mathbb{Z}^+$, and assume $f(m_1, n_1) = f(m_2, n_2)$."