Math 108—Writing Assignment 04

*Due: Friday February 17—2:00*PM

Get the template I made for this assignment. (I also started some proofs for you.) Here's how to do it:

- **Team Member 1:** Go to https://www.sharelatex.com, and make sure you are logged in.
- **Team Member 1:** In a new window, go here:

  https://www.sharelatex.com/project/58a144d46c4adabf41897e03

- **Team Member 1:** Click on the menu icon (upper-left corner - 3 horizontal lines); select "Copy Project"
- **Team Member 1:** When prompted for a name, choose something like "Math 108 - Assignment 04" and click "Copy"
- **Team Member 1:** When this completes you will be back in your own workspace (instead of mine).
- **Team Member 1:** Click on the share icon (upper-right - 5 headed beast). Enter your team member's email address, make sure they "can edit" it, and "Share."
- **Team Member 1 and 2:** After solving the problems (possibly by yourself), work together to make a beautiful write up.
- **Team Member 1 or 2:** Email me (or print and turn in) *one* copy of your final draft.

The problems are below.

1. Let $x, y \in \mathbb{R}$. Prove by contraposition that if xy is irrational, then either x or y is irrational.
2. Let $a, b \in \mathbb{Z}$. Prove that if there exist $m, n \in \mathbb{Z}$ such that $am + bn = 1$, then the only common divisors of a and b are ± 1.

   In other words, assume that there exist $m, n \in \mathbb{Z}$ such that $am + bn = 1$, and prove that for all $c \in \mathbb{Z}$, if c divides both a and b, then $c = \pm 1$.
3. Prove that there does *not* exist a 2×2 matrix A with entries from $\mathbb{R}$ such that $A^2 = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$.

   Hint: try a proof by contradiction. Assume that there does exist some $A = \left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)$ such that $A^2 = \left(\begin{smallmatrix} 0 & 1 \\ 0 & 0 \end{smallmatrix}\right)$. Now compute A^2, and try to find a contradiction.