Math 108—Writing Assignment 09

*Due: Saturday April 15—3:00*PM

Get the template for this assignment. Here's how to do it:

- **Team Member 1:** Go to https://www.sharelatex.com, and make sure you are logged in.
- **Team Member 1:** In a new window, go here:

  https://www.sharelatex.com/project/58ebd8232836187d713d5c87

- **Team Member 1:** Click on the menu icon (upper-left corner - 3 horizontal lines); select "Copy Project"
- **Team Member 1:** When prompted for a name, choose something like "Math 108 - Assignment 08" and click "Copy"
- **Team Member 1:** When this completes you will be back in your own workspace (instead of mine).
- **Team Member 1:** Click on the share icon (upper-right - 5 headed beast). Enter your team member's email address, make sure they "can edit" it, and "Share."
- **Team Member 1 and 2:** After solving the problems (possibly by yourself), work together to make a beautiful write up.
- **Team Member 1 or 2:** Email me (or print and turn in) *one* copy of your final draft.

The problems are below.

1. Let p be a prime number. Prove that for all $\bar{x}, \bar{y}, \bar{z} \in \mathbb{Z}_p$, if $\bar{x}\bar{y} = \bar{x}\bar{z}$ and $\bar{x} \neq \bar{0}$, then $\bar{y} = \bar{z}$.

   Hint: Theorem 3.4.4 could be helpful, but to use it, you will first need to rearrange the equation $\bar{x}\bar{y} = \bar{x}\bar{z}$.

2. Let A and B be sets, and let f be any function from A to B. The *kernel* of f is defined to be the relation on A given by
$$x \, R \, y \iff f(x) = f(y).$$
   Prove that the kernel of f is an equivalence relation on A.

3. For every subset $S \subseteq \mathbb{R}$, define the *characteristic function* of S to be
$$\chi_S(x) = \begin{cases} 1 & \text{if } x \in S, \\ 0 & \text{if } x \notin S. \end{cases}$$
   Prove that for all subsets $A, B \subseteq \mathbb{R}$, $\chi_{A \cup B}(x) = \chi_A(x) + \chi_B(x) - \chi_A(x) \cdot \chi_B(x)$.

   Note that $\chi_{A \cup B}(x)$ is the characteristic function on the set $A \cup B$. You may want to consider using cases. For example: (1) x is in A but not in B, (2) x is in B but not in A, ... (two more cases)...