

About the Course

Languages + Truth in Structures + Proof
(syntax) (semantics)

there are choices (think about your project); we use first-order (predicate) logic

how are these defined?

Overarching Question: what is the relationship b/w truth and proof?

History

Here are some highlights — I'll defer to to book for more details.

- Mid 19th century: math becomes increasing abstract and questions are raised about the foundations of math.
* The time of Boole, De Morgan, Frege, Pierce, Peano, and others
- Paris, 1900, ICM: Hilbert proposes his list of 23 problems for the math community to investigate in the upcoming century.

Hilbert's 2nd Problem:

But above all I wish to designate the following as the most important among the numerous questions which can be asked with regard to the axioms [of arithmetic]: To prove that they are not contradictory, that is, that a finite number of logical steps based upon them can never lead to contradictory results. (Quoted in [Feferman 98])

* Be aware: this statement is open to interpretation.
We will choose one point of view in this course,
but there are others.

• Königsberg, 1930 24-year-old Gödel
announced that

① there is a (mathematical) sentence
that is true but not provable
in a formal system of "classical mathematics"

② no consistent formal system of math
could prove its own consistency!

contains no
contradiction

* often ZFC is adopted as our
formal system, but using it, we
cannot prove that there are no
contradictions (unless there are!)

↪ so, from our point of view, Hilbert's 2nd
problem is hopeless.

Our Goals this semester

- ① Defining Truth and Proof (a deductive system)
- ② Soundness and Completeness of
of Deductive System
- ③ An application: compactness (model theory - yay!)
different notions
- ④ Gödel's Incompleteness Theorems