Math 161—Writing Assignment 09

*Due: Sunday November 04—5:00*PM

Getting Started

1. Get the template for this assignment. Here's how to do it:
   - Go to https://v2.overleaf.com/ (formerly: https://www.sharelatex.com), and **make sure you are logged in**.
   - In a new window, go here:

     https://v2.overleaf.com/read/dsjswyxjcbcc
   - Click on the menu icon in the upper-left and select "Copy Project"
   - When ask for a name, choose something like "Math 161 - WA 09" and click "Copy"
   - When this completes you will be back in your own workspace (instead of mine).
   - After solving the problem(s), type them up using the template.
   - Email me your final draft.
2. Let me know if you have any questions!

Definition. Let $\mathcal{L}_{order} = \{<\}$. A **linear order** is any $\mathcal{L}_{order}$-structure $\mathcal{M}$ that models the following formulas:

- $\lambda_1 :\equiv (x < y) \vee (x = y) \vee (y < x)$;
- $\lambda_2 :\equiv \neg(x < x)$;
- $\lambda_3 :\equiv (x < y) \wedge (y < z) \to (x < z)$.

1. Prove that there does **not** exist a set of $\mathcal{L}_{order}$-formulas Σ such that $\mathcal{M} \models \Sigma$ if and only if $\mathcal{M}$ is a *finite* linear order.

   Hint: argue by contradiction. Assume such a Σ exists; that is, $\mathcal{M} \models \Sigma$ if and only if $\mathcal{M}$ is a finite linear order. Next, enlarge Σ to $\widehat{\Sigma}$ by adding in formulas α_k expressing that a model has at least k elements. Now, like we did in class, use Compactness to show $\widehat{\Sigma}$ has a model, and tie it all together. When you use Compactness, you will need to create some finite linear orders, but remember that you are good at creating models (you did that a lot before).

You will see in the template that I got you started, but please feel free to write it up differently.