EXAM 1 - REVIEW QUESTIONS

LINEAR ALGEBRA

Questions (answers are below)

Examples. For each of the following, either provide a specific example which satisfies the given description, or if no such example exists, briefly explain why not.

(1) (JW) A skew-symmetric matrix A such that the trace of A is 1
(2) (HD) A nonzero singular matrix $A \in M_{2\times 2}$.
(3) (LL) A non-zero 2 x 2 matrix, A, whose determinant is equal to the determinant of the A^{-1}
Note: Chap. 3 is not *on the exam. But this is a* great *one for next time.*
(4) (MS) A 3×3 augmented matrice such that it has infinitely many solutions
(5) (LS) A nonsingular skew-symmetric matrix B.
(6) (BP) A symmetric matrix A, such that $A = A^{-1}$.
(7) (CB) A 2×2 singular matrix with all nonzero entries.
(8) (LB) A lower triangular matrix A in RREF.
(9) (AP) A homogeneous system $Ax = 0$, where $A \in M_{3\times 3}$ and $x \in R^3$, which has only the trivial solution.
(10) (EA) Two $n \times n$ matrices A and B such that $AB \neq BA$
(11) (EB) A nonsingular matrix A such that A^T is singular.
(12) (EF) Three 2×2 matrices A, B, and C such that $AB = AC$ and $A \neq C$.
(13) (LD) A and B are $n \times n$ matrices with no zero entries such that $AB = 0$.
(14) (OH) A matrix $A \in M_{2\times 2}$ where $Ax = 0$ has only the trivial solution.
(15) (AL) An elementary matrix such that $E = E^{-1}$.
(16) (VM) An augmented matrix $[A|b]$ that has no solutions.
(17) (KS) An nonsingular matrix A, such that $Ax = 0$ has more than just the trivial solution.
(18) (TP) An $n \times n$ matrix A that is equal to $(A^T)^T$
(19) (XR) Elementary matrices A and B such that $AB \sim I_n$
(20) (SS) A vector x such that $A^T x = b$ where $A = \begin{bmatrix} 4 & 1 \\ 1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$. Find x.
(21) (JH) An $n \times n$ matrix A which is not equal to $(A^T)^T$.

True or False.

(a) (JW) There exist an $A \in M_{n\times n}$, such that $A \neq 0$, $A \neq I_n$, and $AB = BA$ for all $B \in M_{n\times n}$.
(b) (HD) For all $A, B, C \in M_{2\times 2}$, if $AB = AC$ and $A \neq 0$, then $B = C$.
(c) (LL) Let F be the set of functions with domain equal to the positive real numbers and codomain equal to $\mathbb{R}$. $\forall f \in F\ \exists x \in \mathbb{R}(x = f(x))$
(d) (MS) $A = \begin{bmatrix} 0 & 4 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is an elementary matrix
(e) (LS) $A = \begin{bmatrix} 1 & 2 & 4 & 3 \\ 2 & 1 & 3 & 2 \\ 1 & -2 & 2 & 3 \end{bmatrix}$ is row equivalent to $B = \begin{bmatrix} 2 & 4 & 8 & 6 \\ 1 & -1 & 2 & 3 \\ 4 & -3 & 7 & 8 \end{bmatrix}$.
(f) (BP) $\forall A \in M_{n\times n}(A \sim I_n) \implies A^{-1}$ exists.
(g) (CB) If A is an upper triangular matrix, then A^T is a lower triangular matrix.
(h) (LB) If matrix A is $n \times n$ and A is singular, then the system $Ax = 0$ must have a nontrivial solution.

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(i) (AP) If the linear system $Ax = b$, where $A \in M_{n\times n}$ and $b \in R^n$, has only the trivial solution, then A is a product of elementary matrices.
(j) (EA) Not every nonzero $m \times n$ matrix $A = [a_{ij}]$ is row (column) equivalent to a matrix in row (column) echelon form.
(k) (EB) If we know $AB = I_n$, then to prove $B = A^{-1}$ we must also check that $BA = I_n$.
(l) (LD) The product of two n x n non-singular matrices is non-singular.
(m) (EF) If the product of two matrices is a zero matrix, one of the matrices has to be a zero matrix.
(n) (OH) An invertible matrix A has only one B where B is the inverse of A.
(o) (AL) If A and B are $2x2$ matrices and $AB = O_2$, then $A = O_2$ or $B = O_2$.
(p) (VM) If matrix A has a row/column made up entirely from zeroes, than A is not invertible.
(q) (KS) An $n \times n$ matrix A, which contains a column of zeros, is invertible.
(r) (TP) The matrix $A = \begin{bmatrix} 1 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$ is in RREF form.
(s) (XR) An upper triangular matrix must have a trace of 0.
(t) (SS) $\forall A \in M_{2\times 2}$ if $A \neq 0$ then A is invertible.
(u) (JH) If $AB = I_n$ and $AC = I_n$, then $B = C$.

Answers

Examples - Answers.

(1) (JW) Not possible. Skew-symmetric matrices have all zeros on the main diagonal, so the trace must be zero.
(2) (HD) Let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.
(3) (LL) Any 2 x 2 matrix for which $a_{11}a_{22} - a_{12}a_{21} = 1$
(4) (MS) Let $A = \left[\begin{array}{cc|c} 1 & 7 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$.
(5) (LS) Example: $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.
(6) (BP) $A = I_n$ or $A = -1(I_n)$
(7) (CB) Let $A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$.
(8) (LB) Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.
(9) (AP) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} x = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$
(10) (EA) Let $A = \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 \\ -3 & 4 \end{bmatrix}$. $AB = \begin{bmatrix} -4 & 7 \\ 0 & 5 \end{bmatrix} \neq \begin{bmatrix} -1 & 2 \\ 8 & 2 \end{bmatrix} = BA$.
(11) (EB) Not possible. By Theorem 1.8 we know that if is A is a nonsingular matrix then A^T must also be nonsingular.
(12) (EF) Let $A = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$. Let $B = \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$. Let $C = \begin{bmatrix} 5 & 2 \\ 4 & 7 \end{bmatrix}$. $AB = AC = \begin{bmatrix} 1 & -5 \\ -1 & 5 \end{bmatrix}$. Therefore $AB = AC$ but $B \neq C$.
(13) (LD) $AB = \begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$.
(14) (OH) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ where $ad - bc \neq 0$.
(15) (AL) $E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(16) (VM) $A = \begin{bmatrix} 1 & 0 & 0 & |0 \\ 0 & 1 & 0 & |0 \\ 0 & 0 & 0 & |1 \end{bmatrix}$

(17) (KS) Not possible, by Theorem 2.9 we know that A is invertible if $A\bar{x} = 0$ has only the trivia solution.

(18) (TP) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$. $A^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$. $(A^T)^T = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$.

(19) (XR) Let $A = 2I_n$ and $B = 3I_n$.

(20) (SS) $x = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(21) (JH) By Properties of Transpose (a), the matrix does not exist.

True or False - Answers.

(a) (JW) T, let $A = 2 \cdot I_n$.
(b) (HD) F, See Example 10 on page 39.
(c) (LL) F, let $f(x) = ln(x)$
(d) (MS) False.
(e) (LS) True.
(f) (BP) T
(g) (CB) True.
(h) (LB) True.
(i) (AP) True by Invertibility theorem.
(j) (EA) False. Theorem 2.1 (pg. 89)
(k) (EB) False. Theorem 2.11 (pg 124)
(l) (LD) True (By theorem 1.6)
(m) (EF) False. A counterexample is if $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -8 \\ -1 & 4 \end{bmatrix}$. Then, $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, but neither A nor B is the zero matrix.
(n) (OH) True (Theorem 1.5).
(o) (AL) False.
(p) (VM) True.
(q) (KS) False, by MVP Theorem, if A contains a column of zeros, any product matrix of A and another appropriate sized matrix (such as A^{-1}) would contains a column of zeros, which contradicts the definition of identity matrix.
(r) (TP) True.
(s) (XR) False, for upper triangular matrix, only the lower left part has to be all zeros. The main diagonal can have both zeros or nonzeros.
(t) (SS) False, consider the example $A = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$
(u) (JH) True. If A is invertible, Then A only has a unique inverse.