EXAM 2 - REVIEW QUESTIONS

LINEAR ALGEBRA

Questions (answers are below)

Examples. For each of the following, either provide a specific example which satisfies the given description, or if no such example exists, briefly explain why not.

(1) (JR) A set of vectors $S = \{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_3\}$ which forms a basis for P_3.
(2) (AP) A linearly independent set of at least 5 vectors in $\mathbb{R}^4$.
(3) (BP) A nonsingular matrix $A \in M_{n\times n}$ with nullity$(A) \neq 0$.
(4) (LL) A subspace, W, for the vector space $\mathbb{R}^4$
(5) (TP) An $m \times n$ matrix whose columns rank $>$ its row rank.
Note: rank wil NOT be covered on this exam.
(6) (LS) Let V be the set of all 2×2 matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $abcd = 0$. Create two nonzero matrices B and C showing that V is not closed under addition.
(7) (VM) A matrix $A \in M_{m\times n}$ with nullity$A$ = rank$A$.
(8) (CB) A 3×3 upper triangular matrix whose columns and rows are both linearly independent.
(9) (HD) Two vectors in $\mathbb{R}^2$ that are linearly dependent.
(10) (OH) A 3×3 matrix A where det$A = -1$.
(11) (EB) A matrix A with linearly independent columns for which $\det(A) = 0$.
(12) (XR) A linearly independent subset $S = \{\vec{v}_1, \vec{v}_2, ..., \vec{v}_n\}$ of a vector space V with $\dim V < n$.
(13) (EF) Four vectors in $\mathbb{R}^3$ that are linearly independent.
(14) (SS) A basis $\{v_1, v_2, ..., v_k\}$ of a vector space V where $v_1, v_2, ..., v_k$ are linearly dependent.
(15) (LB) A 2×2 matrix A where $det(A^{-1}) = 0$.
(16) (KS) A $n \times n$ matrix with row rank $>$ column rank.
Note: rank wil NOT be covered on this exam.
(17) (BW) A linearly independent set of vectors $S = \{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_3\}$ such that S spans R^3.
(18) (JH) A singular $n \times n$ matrix with its determinant not equal to zero.
(19) (LD) A basis $S = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ for the vector space of 2×2 matrices with trace equal to zero.
(20) (MS) A nonsingular 3×3 matrix with a row of zeros.
(21) (AL) An $n \times n$ matrix A with $\operatorname{rank} A = n$ and $\det A = 0$.
Note: rank wil NOT be covered on this exam.
(22) (EA) Vector spaces V and W where V is isomorphic to W but W is not isomorphic to V.

True or False.

(a) (JR) It is possible to find a finite set of vectors which spans P, the set of all polynomials.
(b) (AP) Let $A \in M_{n\times n}$ and $x \in \mathbb{R}^n$. If det$A \neq 0$, then the system $Ax = 0$ has only the trivial solution.
(c) (BP) Let S form a basis for $\mathbb{R}^n$. Then S is linearly independent.
(d) (LL) $V = \{x \in \mathbb{R} | x > 0\}$ with operations $\oplus$ as addition and $\odot$ as scalar multipliaction is a real vector space.
(e) (TP) The determinate of an invertible matrix can never equal zero.
(f) (LS) If A and B are $n \times n$ matrices, then $\det(AB) = \det(A)\det(B)$.
(g) (VM) If a set of n vectors S, in an n-dimentional vector space V, spans V, then S is linearly independent.
(h) (HD) If $\mathbf{u} \in V$ where V is a vector space, then $c \odot \mathbf{u} = \mathbf{0}$ means that $\mathbf{u} = \mathbf{0}$

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(i) (CB) Let V be a vector space and S be a subspace of V. Every set that spans S contains a basis for S.

(j) (OH) P_2 is a subspace of P_3.

(k) (EB) If det $(A) = 0$, them A is the product of elementary matrices.

(l) (XR) If $A \in M_{m\times n}$ with $m < n$, then $A\vec{x} = \vec{0}$ has infinitely many solutions.

(m) (EF) If the rows or columns of a square matrix A are linearly independent, then A is invertible.

(n) (SS) If S_1 and S_2 are finite subsets of vector space V and S_1 is a subset of S_2. Then if S_1 is linearly dependent then so is S_2.

(o) (LB) A basis for P_3 can be trimmed to form a basis for P_4.

(p) (KS) An $m \times n$ matrix with $m \neq n$ where the rows and columns of A are both linearly independent.

(q) (BW) If $S = \{\mathbf{v}_1, \mathbf{v}_3, \mathbf{v}_3\}$ is a set of vectors in vector space V and S is linearly dependent, then span S is not a subspace of V.

(r) (LD) The set of all $n \times n$ skew symmetric matrices is not a subspace of M_{nn}.

(s) (JH) If a vector space V has dimension n, then any subset of size n that spans V must be linearly independent.

(t) (MS) If linearly dependent vectors $\{v_1, v_2, \ldots, v_n\}$ span the the vector space V, then the vectors form a basis for V.

(u) (AL) If W is a subspace of the vector space V, then every linear combination of vectors from V is also in W.

(v) (EA) Two finite-dimensional spaces are isomorphic if and only if their dimensions are equal

Answers

Examples - Answers.

(1) (JR) No such example exists because P_n has dimension $n+1$, by a proposition from class. Therefore, a basis for P_3 must contain exactly 4 vectors.

(2) (AP) Impossible. By Theorem 4.10, a linearly independent set of vectors in a vector space V cannot have more elements than the basis for V.

(3) (BP) No such example exists because nullity$(A) \neq 0$ implies that $A\mathbf{x} = \mathbf{0}$ has more than only the trivial solution. That implies, by the Invertibility Theorem, that A is singular.

(4) (LL) W is the set of all vectors of the form $\begin{bmatrix} x \\ 0 \\ y \\ 0 \end{bmatrix}$, where $x, y \in \mathbb{R}$

(5) (TP) No such example exists, by theorem 4.18, row rank = col rank.

(6) (LS) $B = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$.

(7) (VM) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ (rank$A$ = nullity$A = 2$).

(8) (CB) $A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ 0 & a_{22} & a_{23} \\ 0 & 0 & a_{33} \end{bmatrix}$ Where $a_{11}, a_{22}, a_{33} \neq 0$.

(9) (HD) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix} \right\}$ because one is the multiple of the other.

(10) (OH) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$.

(11) (EB) No such example exists, by theorem 4.5.

(12) (XR) There is no such example by Theorem 4.10

(13) (EF) No such example exists because $\mathbb{R}^n$ has dimension n which means that there can be at most n vectors that are linearly independent.

(14) (SS) No such example exists by definition 4.10.

(15) (LB) Impossible. For a matrix to be invertible, $det(A) \neq 0$.

(16) (KS) No such example exists by Theorem 4.18.

(17) (BW) One such example is $\mathbf{v}_1 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, and $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

(18) (JH) No such example exists. By Theorem 3.8, if A is an $n \times n$ matrix, then A is nonsingular if and only if $\det(A) \neq 0$.

(19) (LD) $\mathbf{v}_1 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$.

(20) (MS) Not possible because by Theorem 3.4, if a matrix has a row of zeros then the $\det A = 0$ and the matrix would be singular.

(21) (AL) Not possible. By Corollary 4.8 if A is an $n \times n$ matrix, then rank $A = n$ if and only if $\det(A) \neq 0$.

(22) (EA) Not possible, By Theorem 4.15, if V is isomorphic to W, then W is Isomorphic to V

True or False - Answers.

(a) (JR) False. P is infinite dimensional, so its basis must contain infinite vectors.

(b) (AP) True; by the invertibility theorem, if $\det A \neq 0$, then the homogeneous system $Ax = 0$ has only the trivial solution.

(c) (BP) True, by the definition of a basis.

(d) (LL) False, V is not closed under $\odot$

(e) (TP) True, only singular matrices have determinate = 0.

(f) (LS) True. See Theorem 3.9 (page 153).

(g) (VM) True. According to Theorem 4.12 S has to be a basis for V and thus, by definition 4.10 S is linearly independent.

(h) (HD) False. The scalar c can be 0.

(i) (CB) True, see Theorem 4.9.

(j) (OH) True, Theorem 4.3.

(k) (EB) False, under the invertibility theorem.

(l) (XR) Yes, by theorem 4.19 the nullity of A must be greater than 0 when $m < n$, which means there exists at least one free variable.

(m) (EF) True. By Theorem 4.5, if the columns or rows of a square matrix are linearly independent, then $\det A \neq 0$. If $\det A \neq 0$, A is invertible, by our invertibility theorem.

(n) (SS) True, by Theorem 4.6.

(o) (LB) False, a basis for P_4 will need to have 5 vectors, while a basis for P_3 has 4 vectors.

(p) (KS) False, by Thm 4.18 and proposition 1 of WA#11 which states that if vector space V has dimension n, then any subset of $m > n$ vectors must be linearly dependent.

(q) (BW) False. By Theorem 4.4, we know that span S is a subspace of V because dependence is not taken into account when determining what is a subspace.

(r) (LD) False. See #18(c) in Homework #8

(s) (JH) True. By definition 4.10 and 4.11, any subset of size n that spans V is a basis. So it is linearly independent.

(t) (MS) False, the vectors $\{v_1, v_2, \ldots, v_n\}$ must be linearly independent to be a basis for vector space V.

(u) (AL) False because W is a subspace of V so vectors in V might not be contained in W, nor linear combinations of those vectors.

(v) (EA) True by Theorem 4.16