Linear Algebra
MATH 224W – Spring 2015

Week 11: Dimension and Isomorphisms

Homework #10 **due Thursday, Nov. 5**

§4.6 #2(a)(c), 4(a)(c),10, 11, 13, 20(b), 22, 30

For #13, identify each polynomial $a + bt + ct^2 + d^3$ with the 4-vector $\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$ (or $\begin{bmatrix} d \\ c \\ b \\ a \end{bmatrix}$ - but be consistent), and then follow the same approach as for #11. Make sure that the answer you provide consists of polynomials though (and not 4-vectors).

When asked to "Generalize to M_{mn}" in exercise #30, make sure to describe a basis for M_{mn} **and** give the dimension of M_{mn}.

Writing Assignment #10 **due Monday, Nov. 9**

AP #1 Let $\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ be vectors in a vector space. If $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent and $\mathbf{u} \notin \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$, prove that $\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ are linearly independent.

AP #2 Let $\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ be vectors in a vector space. If $\mathbf{u} \in \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$, prove that

$$\text{span}\{\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\} = \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}.$$

Hint: Let $S = \text{span}\{\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ *and* $T = \text{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$*. To show* $S = T$*, you must show two things:* $S \subseteq T$ *and* $T \subseteq S$*. One of these is easy, and the other requires a little work.*

AP #3 Let V and W be vector spaces, and let $T : V \to W$ be an isomorphism. Prove that if $\{\mathbf{v}_1, \ldots, \mathbf{v}_k\}$ is a linearly independent set of vectors in V, then $\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)\}$ is a linearly independent set of vectors in W.
You can use the results in Exercise 29 on page 269, one of which is that if $T : V \to W$ **is an isomorphism, then** $T(\mathbf{0}) = \mathbf{0}$**.** *Hint: in addition to using the algebraic properties of an isomorphism (see Definition 4.13 (a) and (b)), you will also need to use that* T *is one-to-one.*

AP #4 Let V and W be vector spaces, and let $T : V \to W$ be an isomorphism. Prove that if $\text{span}\{\mathbf{v}_1, \ldots, \mathbf{v}_k\} = V$, then $\text{span}\{T(\mathbf{v}_1), \ldots, T(\mathbf{v}_k)\} = W$.
Hint: in addition to using the algebraic properties of an isomorphism (see Definition 4.13 (a) and (b)), you will also need to use that T *is onto, i.e. that* $\text{range}(T) = W$*.*