

Linear Algebra
MATH 224W – Spring 2015

Week 13: Linear transformations, kernel and range

Homework #12

due Friday, Nov. 20 at 7:00 pm

§4.9 #9, 34, 35, 36

For #9 in §4.9, you can use a computer (<http://www.wolframalpha.com> is one option) to perform your row reduction as long as you clearly state what you have done.

§6.1 #2, 3, 4, 5, 8(c), 11(c), 12(b), 13(b), 15, 28

For #2–5, if a function is linear, you do not need to explain why. However, for each function that is not a linear transformation, you **MUST** explain why it is not.

Writing Assignment #12

due Monday, Nov. 30

AP #1 Let A and B be $m \times n$ matrices. Prove that $\text{rank}(A + B) \leq \text{rank}(A) + \text{rank}(B)$. *Hint: start by showing that the columns of A together with the columns of B span the column space of $(A+B)$.*

AP #2 Let A be an $n \times n$ matrix, and define a function $L : M_{n \times n} \rightarrow M_{n \times n}$ by $L(X) = AX - XA$. Prove that L is a linear transformation.

AP #3 Let V and W be vector spaces, and let $T : V \rightarrow W$ be a linear transformation. Prove that if $\mathbf{v}_1, \dots, \mathbf{v}_k$ are linearly dependent vectors in V , then $T(\mathbf{v}_1), \dots, T(\mathbf{v}_k)$ are linearly dependent vectors in W .