Linear Algebra
MATH 224W – Spring 2015

Week 14: Eigenvalues and eigenvectors

Homework #13 **due Thursday, Dec. 3**

§6.2 #1, 2, 4, 6, 16, 25, 26
For #16 in §6.2, you can use a computer (http://www.wolframalpha.com is one option) to perform your row reduction as long as you clearly state what you have done.

§6.3 #8(a)(b), 10(b), 22(a)(b)

Writing Assignment #13 **due Monday, Dec. 7**

§7.1 #11
Just prove this when A is upper triangular.

AP #1 Let V and W vector spaces with $\dim V = \dim W$, and assume that $L : V \to W$ is a linear transformation. Prove that L is one-to-one if and only if L is onto.
Hint: Range-Kernel Theorem. Remember that there are two things to show for an if and only if statement.

AP #2 Let V be an n-dimensional vector space. Assume that $L : V \to V$ is a linear transformation such that $L(L(\mathbf{v})) = \mathbf{0}$ for every $\mathbf{v} \in V$. Prove that

$$\dim(\operatorname{range} L) \leq \frac{n}{2}.$$

Hint: First show that $\operatorname{range} L \leq \ker L$.

AP #3 Let A be an $n \times n$ matrix. Prove that A and A^T have the same characteristic polynomial and, hence, that A and A^T have the same eigenvalues.

AP #4 Let A be an invertible $n \times n$ matrix, and assume that λ is an eigenvalue of A. Prove that $\lambda \neq 0$, and that λ^{-1} is an eigenvalue of A^{-1}.