Linear Algebra
MATH 224W – Spring 2015

Week 5: Solving linear systems and elementary matrices

Writing Assignment #4 **due Monday, Sept. 21**

§1.6 #21

Pg. 81 #18
Hint: In this problem you are assuming that an equation is true for *every* n-vector $\mathbf{x}$, so in particular it is true for the vectors $\mathbf{e}_1, \mathbf{e}_2, \ldots, \mathbf{e}_n$ where

$$\mathbf{e}_1 = \begin{bmatrix}1\\0\\0\\\vdots\\0\end{bmatrix}, \mathbf{e}_2 = \begin{bmatrix}0\\1\\0\\\vdots\\0\end{bmatrix}, \ldots, \mathbf{e}_n = \begin{bmatrix}0\\0\\0\\\vdots\\1\end{bmatrix}.$$

For example, if you plug $\mathbf{e}_1$ in for $\mathbf{x}$, you get a true statement. What does it tell you?

AP#1 Prove that for all statements p, q, and r,

$$p \implies (q \vee r) \equiv [p \wedge (\sim q)] \implies r.$$

You may prove this using a truth table or by using established logical equivalences. If you use logical equivalences, make sure to **cite them by name**. If you choose to use a truth table, you can learn about making tables in LaTeX here: https://www.sharelatex.com/learn/Tables.

AP #2 Determine if implication is associative. That is, prove or disprove the following: for all statements p, q, and r,

$$(p \implies q) \implies r \equiv p \implies (q \implies r).$$

Extra Credit (*Note: I will not give any hints about extra credit problems.*) Call an $n \times n$ matrix A *nilpotent* if $A^k = 0$ for some positive integer k. Prove that if A is an $n \times n$ nilpotent matrix, then $I - A$ is invertible (where I is the $n \times n$ identity matrix).

Homework #4 **due Thursday, Sept. 24**

§0.1 # 2 (b)(d), 3(a)(c), 4(c)(d), 6(a)(b)(c)(e)(f), 8, 9
The exercises for §0.1 are available in the Documents section of Blackboard.

AP #1 Write down the converse and contrapositive of the following statement.

"If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$."

AP #2 Let $P(x, y)$ denote the formula $x \geq y$. Also, let $\mathbb{N}$ denote the set of natural numbers (nonnegative integers); that is, $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$. Determine whether the following are true or false; justify your answers!

(a) $\forall x, y \in \mathbb{N}[P(x, y)]$
(b) $\forall x \in \mathbb{N}, \exists y \in \mathbb{N}[P(x, y)]$
(c) $\exists x \in \mathbb{N}, \forall y \in \mathbb{N}[P(x, y)]$
(d) $\exists x, y \in \mathbb{N}[P(x, y)]$

AP #3 Write down the negation of each of the following statements in such a way that *negation symbols only appear next to the predicates* p, q, or r.

(a) $\exists x \in \mathbb{R}[(\sim p(x)) \wedge q(x)]$

(b) $\forall x \in \mathbb{R}[p(x) \implies [\exists y \in \mathbb{N}[q(x,y) \wedge r(x,y)]]]$

(c) $\exists x, y \in \mathbb{N}[P(x,y)]$

AP #4 Disjunction and conjunction are associative. That is, for statements p, q, and r,

(a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$, and

(b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$.

Further, disjunction and conjunction distribute over one another. That is, for statements p, q, and r,

(c) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$, and

(d) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.

Prove (c) using a truth table. Each truth table should have 8 rows. You do not need to prove (a) or (d), but you should be aware that they are true so that you can make use of them in the future.