Linear Algebra
MATH 224W – Spring 2016

Week 14-15: Eigenvalues, eigenvectors, and diagonalization

Writing Assignment #13 **due Friday, May. 6**

§7.1 #11 *Just prove this when A is lower triangular, but you can use both results for other problems.*

§7.2 #24 *Hint: try using the* ***definition*** *of diagonalizability.*

AP #1 Let A be an $n \times n$ matrix. Prove that A and A^T have the same characteristic polynomial. (Hence, A and A^T have the same eigenvalues, but you do not need to explicitly prove this.)

AP #2 Let A be an invertible $n \times n$ matrix, and assume that λ is an eigenvalue of A. Prove that $\lambda \neq 0$ and that λ^{-1} is an eigenvalue of A^{-1}. *Hint: try using the* ***definition*** *of eigenvalue.*

Extra Credit: Let $A \in M_{n\times n}$, and let $p(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + \cdots + a_1\lambda + a_0$ be the characteristic polynomial of A. Prove that $a_0 = (-1)^n \det(A)$ and $a_{n-1} = -\operatorname{tr}(A)$.

For Fun: *(Not to be turned in)* Using the previous extra credit problem, you can deduce the following (interesting) formulas for determinant and trace...

Let $A \in M_{n\times n}$, and let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of A, including repetitions. (It may be that $\lambda_1, \ldots, \lambda_n$ are complex numbers.) Prove that $\det(A) = \lambda_1 \cdots \lambda_n$ and $\operatorname{tr}(A) = \lambda_1 + \cdots + \lambda_n$.

Homework #13 **due Friday, May. 6**

§7.1 #1, 3, 6, 8(a)(c), 13, 18(b)
For #1,3, and 6, you are asked to find just one eigenvector associated to each eigenvalue.
For #3, you can use any basis for P_2 that you want. Remember that the eigenvectors for this problem should be polynomials.

§7.2 #6, 10(a)(b), 11(a)(c), 16(b), 19
Hint: Theorem 7.5 is very helpful for several of the parts of #6. Show ***all*** *of your work (except for the actual row reduction) for these problems, especially for #19.*

Extra Problems **Not To Be Turned In—But May Appear on the Final**

#1 Prove or disprove the following statement.

> "For every positive integer n, if A and B are invertible $n \times n$ matrices with the same characteristic polynomial, then A and B are similar."

#2 Let $c \in \mathbb{R}$, and let A be an *upper triangular* $n \times n$ matrix such that every entry on the main diagonal is c.
(1) Prove that A is diagonalizable if and only if the nullity of $(cI - A)$ is n.
(2) Prove that A is diagonalizable if and only if A is a diagonal matrix.
Hint: use part 1 to prove part 2. What does it mean if an $n \times n$ matrix has nullity equal to n?