**Linear Algebra**
**MATH 224W – Spring 2016**

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Week 4: logic and proof methods

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**Writing Assignment #3** **due Monday, Feb. 8**

§1.5 #22(b), 50, 51, 54
**All** of your proofs for §1.5 should be **matrix-level** and not entry-level. Make use of the theorems in sections 1.4 and 1.5! Each of the write-ups should be quite short, but **make sure to cite** all of the theorems that you are using.

AP #1 Prove Theorem 1.2(b).
You will probably have to work with the entries of the matrix and make use of summation notation properties.

AP #2 Show that if $A$ is an $n \times n$ matrix with a column of zeros, then $A$ is not invertible.
*Hint:* argue by contradiction. Assume $A$ is invertible. Then there must be an $n \times n$ matrix $B$ such that $BA = I$. Now explain why this is impossible by using a result you proved on the previous writing assignment.

**Homework #3** **due Thursday, Feb. 11**

§1.6 #6, 8, 10, 12, 19
For #19(c), the "$T(u)$" may be confusing; ignore it. You want to find the smallest positive $k$ such that $A^k\mathbf{u} = \mathbf{u}$ for all $\mathbf{u} \in \mathbb{R}^2$.

§0.1 # 2 (b)(d), 3(a)(c), 4(c)(d), 6(a)(b)(c)(e)(f), 8, 9
*The exercises for §0.1 are available in the Documents section of Blackboard.*

AP #1 Write down the converse and contrapositive of the following statement.

"If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} a_n = 0$."

AP #2 Disjunction and conjunction are associative. That is, for statements $p$, $q$, and $r$,
(a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$, and
(b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$.
Further, disjunction and conjunction distribute over one another. That is, for statements $p$, $q$, and $r$,
(c) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$, and
(d) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$.
**Prove (c), and only (c), using a truth table**. Each truth table should have 8 rows. You do not need to prove (a), (b), or (d), but you should be aware that they are true so that you can make use of them in the future.