Linear Algebra
MATH 224W – Spring 2016

Week 9: Span and Linear Independence

Writing Assignment #8 **due ~~Monday, Mar. 28~~ Wednesday, Mar. 30**

AP #1 Let $W = \{A \in M_{2\times 2} |\ \mathrm{tr}(A) = 0\}$. (Recall that we already proved that W is a vector space; in fact, we proved it is a subspace of $M_{2\times 2}$.) Prove that W is spanned by the set

$$S = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}.$$

AP #2 Let $\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k$ be vectors in a vector space V. If $\mathbf{u} \in \mathrm{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$, prove that

$$\mathrm{span}\{\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\} = \mathrm{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}.$$

Hint: Let $S = \mathrm{span}\{\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ *and* $T = \mathrm{span}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$. *To show* $S = T$, *you must show two things:* $S \subseteq T$ *and* $T \subseteq S$. *To show* $S \subseteq T$, *let* s *be an arbitrary element of* S, *and then work to show that* $s \in T$. *To do this, use the definition of* S *to write* $s = c_0\mathbf{u} + c_1\mathbf{v}_1 + \cdots + c_k\mathbf{v}_k$ *for some* $c_0, c_1, \ldots, c_k \in \mathbb{R}$. *Now do some math to show that* $s \in T$. *You can then use a similar approach to show* $T \subseteq S$, *but this should be much easier.*

AP #3 Let $\mathbf{v}_1, \ldots, \mathbf{v}_n$ be vectors in a vector space V, and let $c_1, \ldots, c_n \in \mathbb{R}$ be scalars. Prove that if $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n = 0$ and at least one of $c_1, \ldots, c_n$ is nonzero, then one of the vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ is a linear combination of the remaining vectors.
Hint: begin your proof with "assume that $c_1\mathbf{v}_1 + \cdots + c_n\mathbf{v}_n = 0$ *and that* $c_i \neq 0$ *for some* $1 \leq i \leq n$*." Now try to show that* v_i *is a linear combination of* $\mathbf{v}_1, \ldots, \mathbf{v}_{i-1}, \mathbf{v}_{i+1}, \ldots, \mathbf{v}_n$.

Homework #8 **due ~~Thursday, Mar. 31~~ Friday, Apr. 1**

§4.4 #2, 4(a), 6(a)(d), 8(a)(c), 12, 14
Look at #13 for inspiration on #14.

§4.5 #2, 4, 16