02 – Rational Functions & Vertical Astymptotes

Definition: Rational Functions

We call $f(x)$ a **rational function** if $f(x)$ can be written in the form $f(x) = \dfrac{p(x)}{q(x)}$ for some polynomials $p(x)$ and $q(x)$.

1. Which of the following are rational functions? Justify your answers.

(a) $f(x) = \dfrac{2}{1-x^3}$

(b) $g(x) = 2 + \dfrac{7}{x}$

(c) $h(x) = \dfrac{7x+\sqrt{x}}{x^3+x-1}$

(d) $r(x) = 5x^3 + 3x - \sqrt{2}$

Definition: Limiting behavior

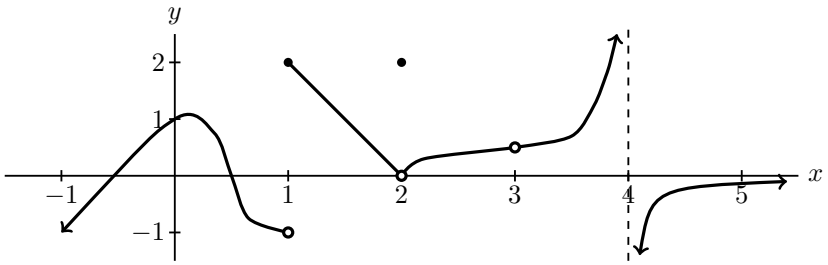
1. $x \to c^+$ means that "x approaches the value c from the right (but does not equal c)"

- For example, $x \to 5^+$ means we are considering x-values like $5.5, 5.1, 5.01, 5.001, \ldots$

2. $x \to c^-$ means that "x approaches the value c from the left (but does not equal c)"

- For example, $x \to 5^-$ means we are considering x-values like $4.5, 4.9, 4.99, 4.999, \ldots$

2. Suppose the graph of a function $f(x)$ is given below. Fill in the blanks.



(a) As $x \to 1^-$, $f(x) \to$ _____

(b) As $x \to 1^+$, $f(x) \to$ _____

(c) As $x \to 2^-$, $f(x) \to$ _____

(d) As $x \to 2^+$, $f(x) \to$ _____

(e) As $x \to 4^-$, $f(x) \to$ _____

(f) As $x \to 4^+$, $f(x) \to$ _____

Definition: Vertical Asymptote

A line $x = c$ is a **vertical asymptote** of $y = f(x)$ if *at least one* of the following are true:

- As $x \to c^+$, $f(x) \to \infty$
- As $x \to c^+$, $f(x) \to -\infty$
- As $x \to c^-$, $f(x) \to \infty$
- As $x \to c^-$, $f(x) \to -\infty$

3. What are the vertical asymptotes of the graph of $y = f(x)$ in the previous example?

4. Let $f(x) = \dfrac{2x}{x-3}$. Answer the following by plugging in x-values closer and closer to 3.

(a) As $x \to 3^-$, $f(x) \to$ _____

(b) As $x \to 3^+$, $f(x) \to$ _____

(c) Is the line $x = 3$ an asymptote of the graph $y = f(x)$. Why or why not?

Strategy: Finding Vertical Asymptotes of Rational functions

The line $x = c$ is a vertical asymptote of the rational function $f(x) = \dfrac{p(x)}{q(x)}$ if $q(c) = 0$ and $p(c) \neq 0$.
If both $q(c) = 0$ and $p(c) = 0$, then factor, cancel common factors, and check again.

5. Find all vertical asymptotes of each of the following rational functions.

(a) $f(x) = \dfrac{x+3}{2x-7}$

(b) $g(x) = \dfrac{x^2+5x+6}{x+2}$

(c) $h(x) = \dfrac{x-5}{2x^2+4x-3}$