06 – Exponential Functions

Definition: Exponential Functions

We call $f(x)$ an **exponential function** if $f(x)$ can be written in the form $f(x) = b^x$ for some positive constant b. The number b is called the **base**.

1. Determine which of the following are exponential functions.

(a) x^3

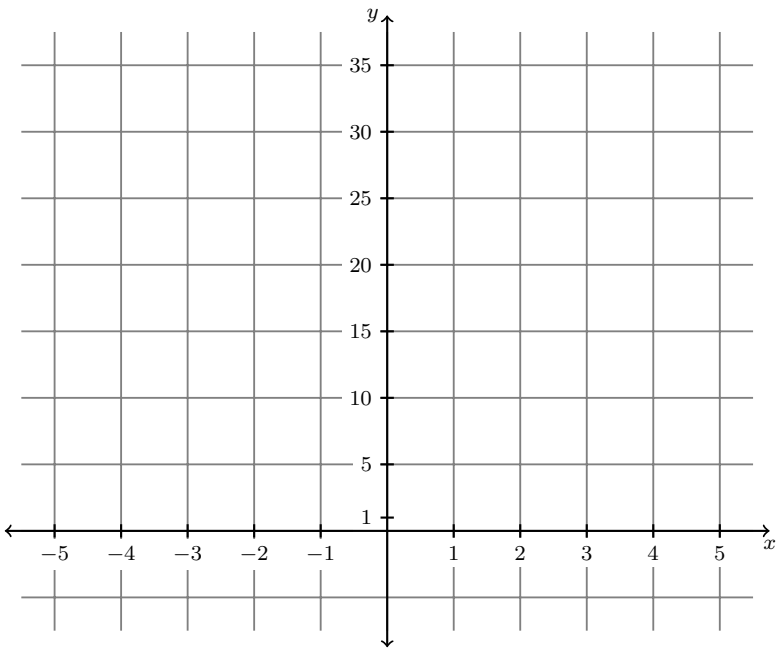
(b) 3^x

(c) x^x

(d) $\left(\frac{2}{\pi}\right)^x$

2. Consider the exponential functions 2^x and $\left(\frac{1}{2}\right)^x$. Fill in the following table of values, and then sketch the graph of each function.

x	2^x	$\left(\frac{1}{2}\right)^x$
4		
3		
2		
1		
0		
-1		
-2		
-3		
-4		



3. Use your work above to describe the domain, range, and end behavior of each of the following functions. Are there any asymptotes?

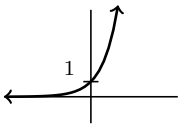
(a) $f(x) = 2^x$

(b) $g(x) = \left(\frac{1}{2}\right)^x$

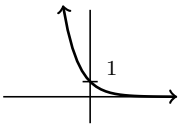
Theorem: Shape of exponential graphs

Consider the exponential function b^x.

- If $b > 1$, then b^x is increasing. As $x \to \infty$, $f(x) \to \infty$. As $x \to -\infty$, $f(x) \to 0$.
- If $0 < b < 1$, then b^x is decreasing. As $x \to \infty$, $f(x) \to 0$. As $x \to -\infty$, $f(x) \to \infty$.



$b > 1$

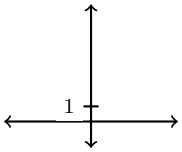


$0 < b < 1$

Definition: Base e

The constant e is defined to be the limiting value of $\left(1 + \frac{1}{x}\right)^x$ as $x \to \infty$. It is approximately $2.71828\ldots$

4. Use the fact that e is close to 2 to sketch a graph of e^x. Is it increasing or decreasing? What is the end behavior?



5. Graph both of the following functions, and plot 3 points on each graph. Determine the range of each and all asymptotes.

(a) $f(x) = 4^{x+1} + 1$

(b) $g(x) = -\left(\frac{1}{3}\right)^x - 2$