07 – Modeling with exponential functions

1. In 2010, the Canadian population was approximately 34 million people. The annual population growth rate is about 0.8%. Let $P(t)$ be the function that gives the population t years after 2010.

(a) Explain why the population next year is 100.8% of what it was this year.

(b) Fill in the table.

$P(0)$	$P(1)$	$P(2)$	$P(3)$

(c) Write a formula for $P(t)$.

(d) Use your formula to find the population in 2030.

2. Suppose that a forest covers an area of 5000 km^2 in 2019. Assume that $\frac{1}{20}$ of the remaining forest area is removed each year. Let $F(t)$ be a function that gives the area of the forest t years after 2019.

(a) If $\frac{1}{20}$ of the area is removed each year, what fraction of it remains?

(b) Fill in the table.

$F(0)$	$F(1)$	$F(2)$	$F(3)$

(c) Write a formula for $F(t)$.

(d) Use your formula to find the amount of forest remaining in 2035.

3. Suppose that \$8000 is invested at a yearly interest rate of 3.5%. Suppose the interest is compounded annually. Let $A(t)$ be the amount of money after t years.

(a) Fill in the table.

$A(0)$	$A(1)$	$A(2)$	$A(3)$

(b) Write a formula for $A(t)$.

(c) Use your formula to find the amount of money after 10 years.

Theorem: Compounding interest

Suppose that an amount M is invested, and it increases by a factor of r (or $100r$%) each year. Let $A(t)$ be the amount of money after t years.

- If the interest is compounded n times each year, then $A(t) = M\left(1 + \frac{r}{n}\right)^{nt}$.
- If the interest is compounded "continuously", then $A(t) = Me^{rt}$.

4. Suppose that \$8000 is invested at a yearly interest rate of 3.5%. Let $A(t)$ be the amount of money after t years.

(a) Assuming that the interest is compounded quarterly, find a formula for $A(t)$, and use it to find the amount of money after 10 years.

(b) Assuming that the interest is compounded continuously, find a formula for $A(t)$, and use it to find the amount of money after 10 years.