08 – Logarithmic Functions

Definition: Logarithmic Functions

The function $\log_b x$ is the *inverse function* of the exponential function b^x. A function of the form $\log_b x$ is called a **logarithmic function**. Because $\log_b x$ is the inverse of b^x, we have that

$$y = \log_b x \iff b^y = x.$$

We usually write $\ln x$ in place of $\log_e x$ and $\log x$ in place of $\log_{10} x$.

1. Rewrite each logarithmic equation as an equivalent exponential equation, and vice versa.

(a) $2 = \log_9 81$

(b) $\ln 1 = 0$

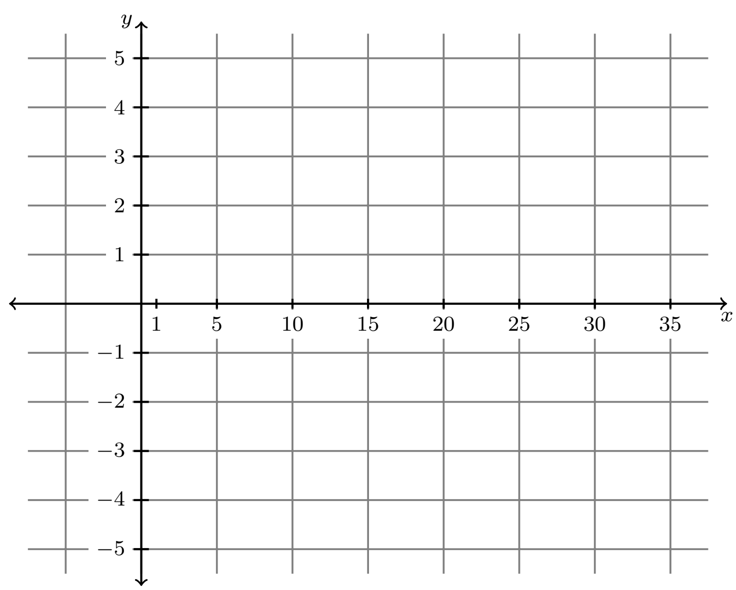
(c) $10^x = \frac{1}{100}$

(d) $e^3 = y$

2. Fill in the following table of values for the exponential function 2^x and use it to find a table of values for $\log_2 x$. Then sketch the graph of $\log_2 x$.

x	2^x
4	
3	
2	
1	
0	
-1	
-2	
-3	
-4	

x	$\log_2 x$

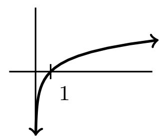


3. Use your work above to describe the domain, range, and end behavior of $\log_2 x$. Are there asymptotes?

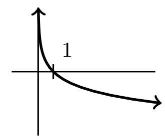
Theorem: Shape of logarithmic graphs

Consider the logarithmic function $\log_b x$. The domain is $(0, \infty)$.

- If $b > 1$, then $\log_b x$ is increasing. As $x \to \infty$, $\log_b x \to \infty$. As $x \to 0^+$, $\log_b x \to -\infty$.
- If $0 < b < 1$, then $\log_b x$ is decreasing. As $x \to \infty$, $\log_b x \to -\infty$. As $x \to 0^+$, $\log_b x \to \infty$.

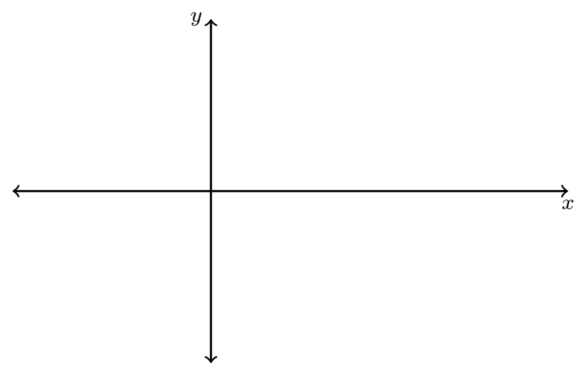


$b > 1$



$0 < b < 1$

4. Graph the function $f(x) = -\ln(x + 2) - 1$. Determine the domain and all asymptotes.



5. Determine the domain of each of the following.

(a) $g(x) = 7 + \log(25 - x^2)$.

(b) $h(x) = \dfrac{13}{1 - \ln x}$.

6. Simplify each expression.

(a) $\log_3 81$

(b) $\log_2 \left(\frac{1}{32}\right)$

(c) $\ln \left(\frac{1}{e^2}\right)$

7. If $5 = 2^{3x-2}$, what is x?