09 – Properties of Logarithms

Theorem: Basic properties of logarithms

- $\log_b(b^x) = x$
- $b^{\log_b(x)} = x$
- $\log_b 1 = 0$
- $\log_b b = 1$

Theorem: Algebraic properties of logarithms

- $\log_b(xy) = \log_b(x) + \log_b(y)$
- $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$
- $\log_b(x^p) = p\log_b(x)$

1. Expand each logarithm into a sum or difference of logarithms of just a single variable, with no powers.

(a) $\log_2(zy^3)$

(b) $\log\left(\dfrac{x}{yz}\right)$

(c) $\log_5\left(\dfrac{25x}{z^2\sqrt{y}}\right)$

2. Rewrite each expression as a single logarithm, with no number in front.

(a) $2\ln x + 3\ln y - \ln z$

(b) $5\ln(x) - \dfrac{1}{3}\ln y - 2\ln(xy)$

3. Determine if each statement is true or false.

(a) $\log\left(\dfrac{x+y}{z}\right) = \log x + \log y - \log z$

(b) $\ln\left(\dfrac{1}{\sqrt{x^2+y^2}}\right) = -\dfrac{1}{2}\ln\left(x^2+y^2\right)$

4. Simplify each expression as much as possible by using rules of logs to expand.

(a) $\log_2\left(\sqrt[4]{y\sqrt{2}}\right)$

(b) $\ln\left(\dfrac{2x(x^2+3)^8}{\sqrt{4-3x}}\right)$

Theorem: Change of base formula

$$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$$

5. Use your calculator to compute each of the following.

(a) $\log_2 153$

(b) $\log_{\frac{1}{3}} 100$

6. If $3^{1+x^2} = 100$, what are the possible values for x? Round to the nearest tenth, if needed.