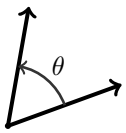
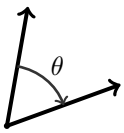
12 – Measuring Angles

Definition: Angles and units of measure

An **angle** is formed by rotating a ray about its endpoint. The measure of the angle is *positive if the rotation is counterclockwise*—the measure is *negative if the rotation is clockwise.*



θ is positive



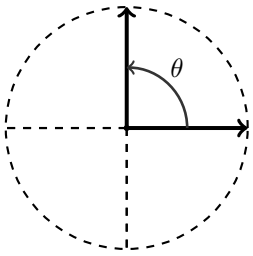
θ is negative

Degrees: An angle corresponding to a *full rotation* is 360 **degrees** (usually written as 360°).

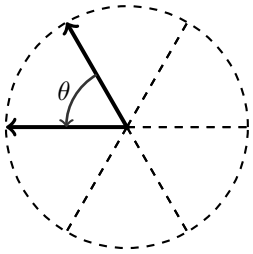
Radians: An angle corresponding to a *full rotation* is 2π **radians** (usually written as 2π).

1. Determine the measure of each of the angles in both degrees and radians. The dashed lines are just to help you identify how much rotation has occurred. *Explain your answers.*

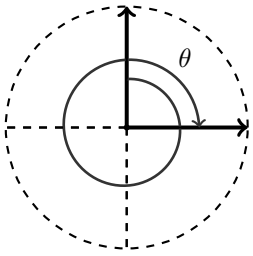
(a)



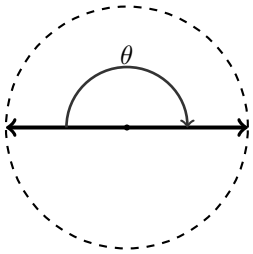
(c)



(b)

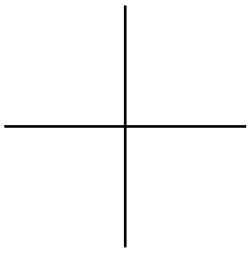


(d)

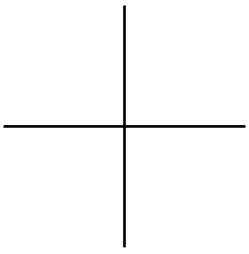


2. Draw angles of each measure. Draw them in *standard form* by putting the vertex at the origin and the initial (starting) side of the angle on the positive x-axis.

(a) $-270°$



(b) $\frac{\pi}{8}$



Definition: Coterminal angles

Two angles are **coterminal** if they have the same initial side and same terminal side.

Example: $60°$ and $780°$ are coterminal angles

The following two angles are coterminal.

3. Find an angle coterminal to $\theta = 400°$ in the range $0°$ to $360°$.

4. Find an angle coterminal to $\theta = -\dfrac{10\pi}{3}$ in the range 0 to 2π.

Theorem: Converting between degrees and radians

From degree to radians: If θ is in degrees, multiply it by _____ to convert it to radians.

From radians to degrees: If θ is in radians, multiply it by _____ to convert it to degrees.

5. If the angle is in degrees, express it in radians, and vice versa.

(a) $75°$

(b) $-210°$

(c) $-\dfrac{13\pi}{6}$ radians

(d) 2 radians