13 – Unit Circle & Trigonometric Functions

Definition: Unit Circle

The **unit circle** is the circle of radius 1 with center at the origin. A point (x, y) is on the unit circle precisely when $x^2 + y^2 = 1$.

1. Determine if each point is on the unit circle or not.

(a) $\left(\frac{3}{5}, \frac{4}{5}\right)$

(b) $(1, 1)$

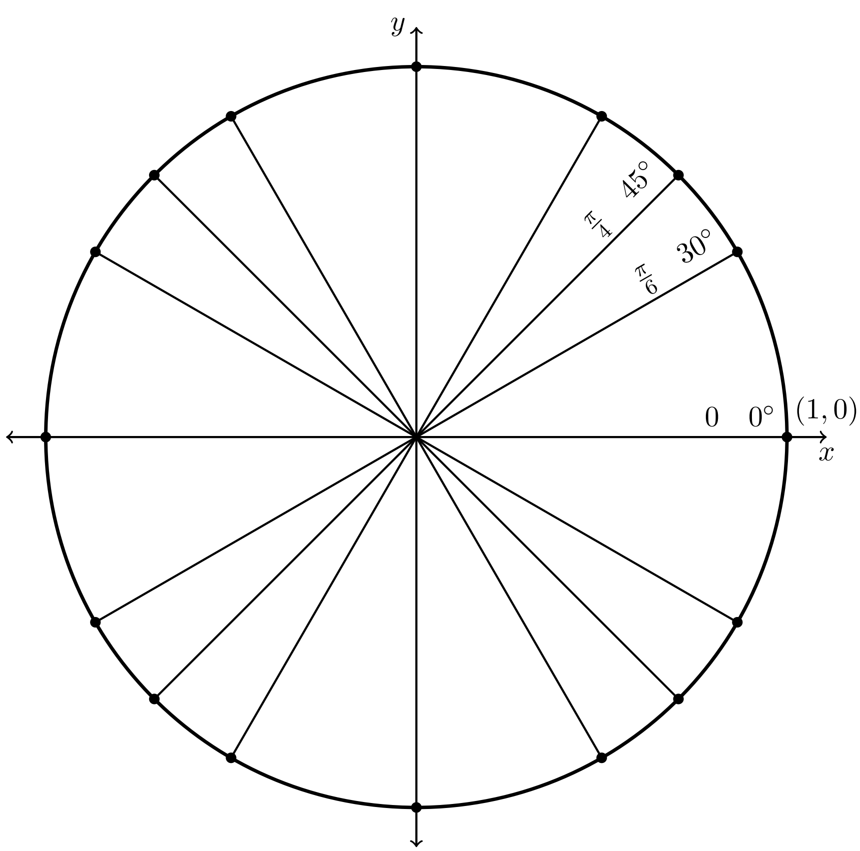
(c) $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

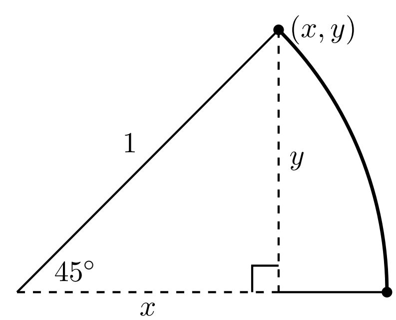
(d) $(0, -1)$

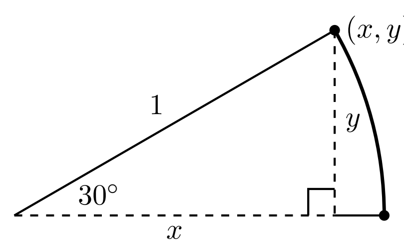
2. Label each spoke in the unit circle below with its angle.

3. Label each point on the unit circle in the first quadrant. Use the triangles off to the side to help you.

4. Use the points you found in the first quadrant to help label the other points on the unit circle.







Definition: Trigonometric Functions

Let θ be an angle in radians, and let (x, y) be the point on the unit circle corresponding to θ. Then

- $\sin\theta = y$
- $\cos\theta = x$
- $\tan\theta = \frac{y}{x}$ (if $x \neq 0$)
- $\csc\theta = \frac{1}{y}$ (if $y \neq 0$)
- $\sec\theta = \frac{1}{x}$ (if $x \neq 0$)
- $\cot\theta = \frac{x}{y}$ (if $y \neq 0$)

5. Let θ be the angle corresponding to the point $\left(\frac{3}{5}, \frac{4}{5}\right)$, which is on the unit circle. Compute $\sec\theta$.

6. Use your unit circle on the front to compute sine, cosine, and tangent of each of the the following.

(a) $135°$

(b) $-210°$

(c) $\frac{5\pi}{6}$

(d) $\frac{7\pi}{2}$

7. Suppose you know that $\sin(t) = 0.7$. Compute each of the following.

(a) $2\sin(t)$

(b) $\sin^2(t)$

(c) $\sin(t + 2\pi)$

(d) $\sin(-t)$

Theorem: Periodic and even/odd properties of sine and cosine

- $\sin(t + 2\pi) =$ __________
- $\sin(-t) =$ __________

  $\sin t$ is an __________ function
- $\cos(t + 2\pi) =$ __________
- $\cos(-t) =$ __________

  $\cos t$ is an __________ function