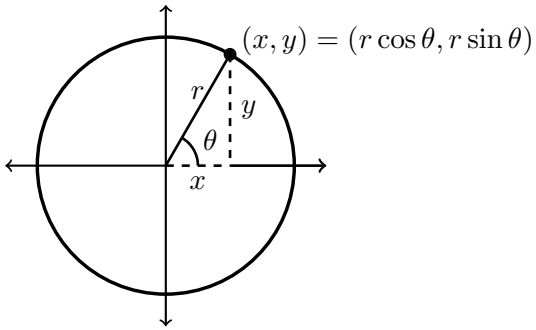
14 – Trig. with Triangles & Fundamental Properties

Theorem: Trigonometric functions using circles of radius r

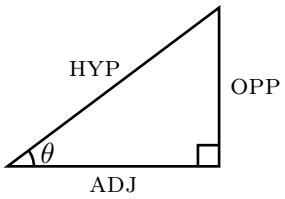
If (x, y) is a point corresponding to θ on a circle of radius r centered at the origin, then $x = r\cos\theta$ and $y = r\sin\theta$.



- $\sin\theta = \dfrac{y}{r}$
- $\cos\theta = \dfrac{x}{r}$
- $\tan\theta = \dfrac{y}{x}$
- $\csc\theta = \dfrac{r}{y}$
- $\sec\theta = \dfrac{r}{x}$
- $\cot\theta = \dfrac{x}{y}$

Theorem: Trigonometric functions using triangles

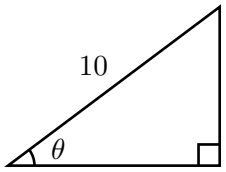
Let θ be an acute angle in a right triangle. The trigonometric functions can be determined in terms of the ADJACENT side, OPPOSITE side, and HYPOTENUSE as follows.



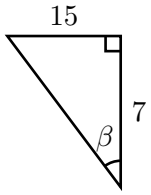
- $\sin\theta = \dfrac{\text{OPP}}{\text{HYP}}$
- $\cos\theta = \dfrac{\text{ADJ}}{\text{HYP}}$
- $\tan\theta = \dfrac{\text{OPP}}{\text{ADJ}}$
- $\csc\theta = \dfrac{\text{HYP}}{\text{OPP}}$
- $\sec\theta = \dfrac{\text{HYP}}{\text{ADJ}}$
- $\cot\theta = \dfrac{\text{ADJ}}{\text{OPP}}$

1. Compute sine, cosine, and tangent of each angle below.

(a)

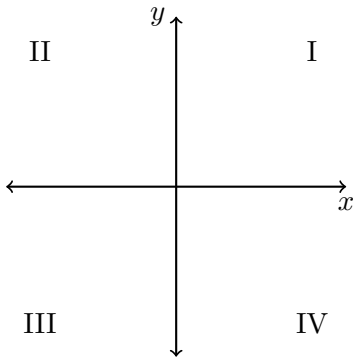


(b)



2. A 5.5 foot-tall person is standing on on a 200 foot cliff next to the ocean. The person sees a sailboat and estimates that angle of depression (tilting their head downward from looking straight ahead to looking at the sailboat) is about 30°. How far off the the coast is the sailboat?

3. In each of the four quadrants, label if each of sine, cosine, and tangent are positive or negative .



Theorem: Trigonometric functions in terms of sine and cosine

- $\csc\theta =$
- $\tan\theta =$
- $\sec\theta =$
- $\cot\theta =$

Theorem: Pythagorean identities

- $\sin^2\theta + \cos^2\theta = 1$
- $\tan^2\theta + 1 = \sec^2\theta$

4. Suppose that you don't know θ, but you do know that θ is in the fourth quadrant and $\cos\theta = \dfrac{\sqrt{7}}{3}$.

(a) Use a Pythagorean identity to find $\sin\theta$.

(b) Find the values of all of the trigonometric functions at θ.

5. The point $(-2, -\sqrt{5})$ is on the terminal side of the angle θ. (See below.) Find the values of all of the trigonometric functions at θ.