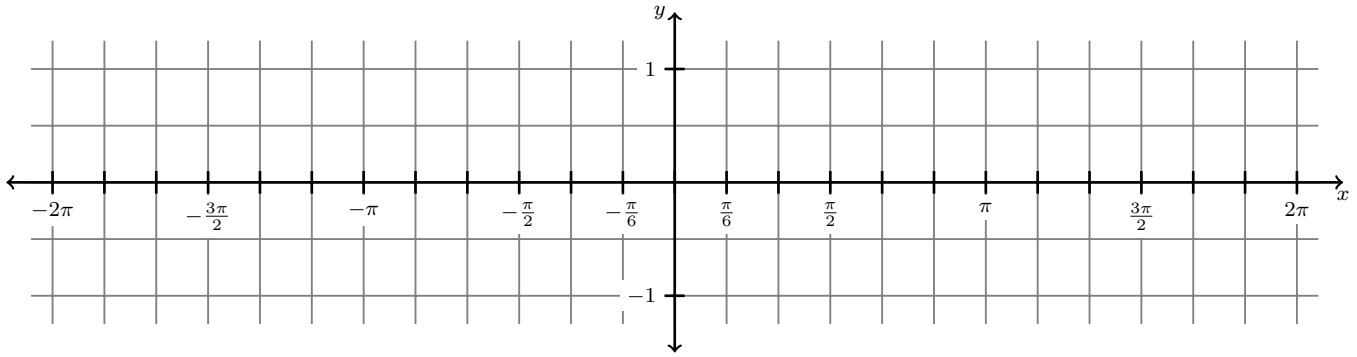
15 – Graphing Sine and Cosine

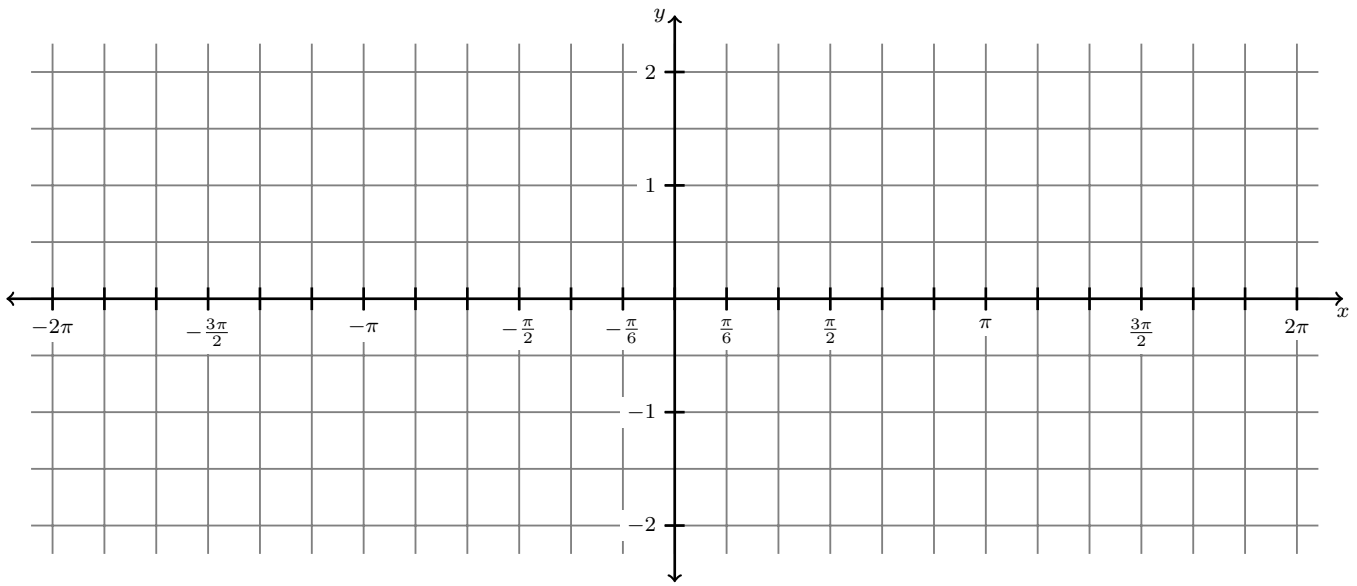
1. Use your unit circle to fill in the following table of values for $\sin x$. Then plot each of the corresponding points, and use them to sketch the graph of $\sin x$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{11\pi}{6}$	2π
$\sin x$													



2. Fill in the table of values for each function below. You can use a calculator if needed.

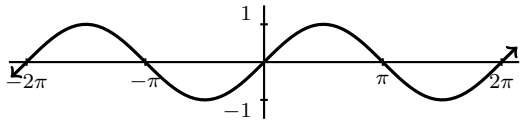
x	$-\pi$	$-\frac{5\pi}{6}$	$-\frac{2\pi}{3}$	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
$2\sin x$													
$\sin(2x)$													



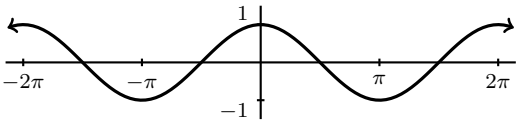
3. What is the *amplitude* and *period* of each of the functions in the previous problem?

Theorem: Shape of $\sin x$ and $\cos x$

The functions $\sin x$ and $\cos x$ have domain $(-\infty, \infty)$, range $[-1, 1]$, and a period of 2π.



$\sin x$



$\cos x$

Theorem: Graphing sinusoidal functions

Suppose you want to graph

$$y = A\sin(Bx - C) + D \qquad \text{or} \qquad y = A\cos(Bx - C) + D$$

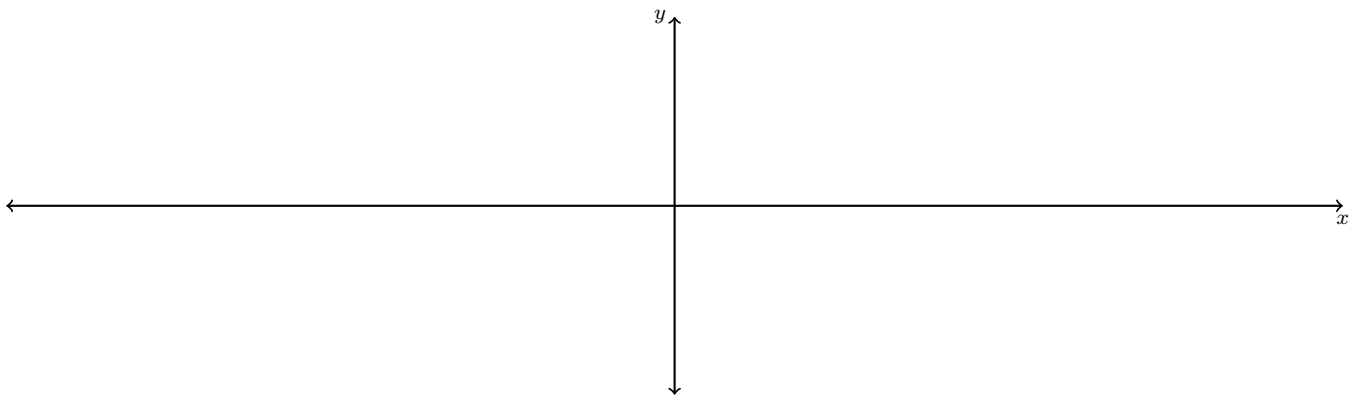
- The amplitude is $|A|$.
- The period is $\frac{2\pi}{B}$.
- The vertical shift is D.
- The phase shift (horizontal shift) is $\frac{C}{B}$.

4. Find the amplitude, period, phase shift, and vertical shift of each of the following.

(a) $f(x) = 2\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right) + 1$

(b) $g(x) = -7\sin\left(\frac{\pi}{2}x + \pi\right) - 3$

5. Graph each of $y = 2\cos\left(\frac{1}{2}x\right)$ and $y = 2\cos\left(\frac{1}{2}x - \frac{\pi}{4}\right) + 1$ below. Draw at least one full period, and label several points.



6. An object oscillating up and down on a spring is moving in *simple harmonic motion*, so the height of the object at time t can be modeled by a function of the form $f(t) = A\sin(Bt - C)$. Suppose that at time $t = 0$ an object attached to a spring is at height 0 ft and is moving downwards. If the period of the oscillations is 5 seconds and the amplitude is 1.7 ft, write an equation of the form $f(t) = A\sin(Bt - C)$ to model the height at time t in seconds.