17 – Inverse Trigonometric Functions

Definition: Inverse Trigonometric Functions

The **inverse trigonometric functions** are defined as follows. Note the restrictions on the domains.

- $\sin^{-1} x$ is the inverse of $\sin x$ on $[-\frac{\pi}{2}, \frac{\pi}{2}]$. It has domain $[-1, 1]$ and range $[-\frac{\pi}{2}, \frac{\pi}{2}]$.
  - If $y = \sin^{-1} x$, then y is the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ such that $\sin y = x$.
- $\cos^{-1} x$ is the inverse of $\cos x$ on $[0, \pi]$. It has domain $[-1, 1]$ and range $[0, \pi]$.
  - If $y = \cos^{-1} x$, then y is the angle between 0 and π such that $\cos y = x$.
- $\tan^{-1} x$ is the inverse of $\tan x$ on $(-\frac{\pi}{2}, \frac{\pi}{2})$. It has domain $(-\infty, \infty)$ and range $(-\frac{\pi}{2}, \frac{\pi}{2})$.
  - If $y = \tan^{-1} x$, then y is the angle between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ such that $\tan y = x$.

We often write $\arcsin x$ in place of $\sin^{-1} x$ with similar "arc" notation for the other inverse trig. functions too. *See the book for the definition of the remaining three inverse trig. functions.*

1. Use your unit circle to find the exact value of each of the following (in radians).

(a) $\sin^{-1}(\frac{1}{2})$

(b) $\cos^{-1}(0)$

(c) $\arccos(-\frac{\sqrt{3}}{2})$

(d) $\arcsin(-\frac{\sqrt{3}}{2})$

(e) $\sin^{-1}(-1)$

(f) $\arctan(-1)$

2. Solve each equation for θ. Give your answers in degrees. You can use calculator.

(a) $7\sin\theta = 3$ and θ is in quadrant I

(b) $7\sin\theta = 3$ and θ is in quadrant II

3. Find the exact value of each of the following without using a calculator.

(a) $\cos(\arccos(-0.15))$ **(b)** $\tan(\tan^{-1}(13))$

4. Find the exact value of each of the following without using a calculator.

(a) $\arccos(\cos(70^\circ))$ **(b)** $\arccos(\cos(200^\circ))$ **(c)** $\arcsin(\sin(350^\circ))$

5. Find the exact value of each of the following without using a calculator.

(a) $\tan(\sin^{-1}(\frac{3}{5}))$ **(b)** $\sec(\tan^{-1}(\frac{2}{7}))$

6. Suppose you notice that a 3 foot tall kid is casting a 4 foot shadow. Can you determine the angle of elevation of the sun?