18 – Sum, Difference, Double-Angle, & Half-Angle Formulas

Theorem: Sum and Difference Formulas

- $\sin(u+v) = \sin u \cos v + \cos u \sin v$
- $\sin(u-v) = \sin u \cos v - \cos u \sin v$
- $\cos(u+v) = \cos u \cos v - \sin u \sin v$
- $\cos(u-v) = \cos u \cos v + \sin u \sin v$
- $\tan(u+v) = \dfrac{\tan u + \tan v}{1 - \tan u \tan v}$
- $\tan(u-v) = \dfrac{\tan u - \tan v}{1 + \tan u \tan v}$

1. Write $165°$ as a sum or difference of angles represented on the unit circle. Then find $\cos(165°)$ exactly.

2. Find the *exact* value of each of the following. *Do not give a decimal approximation.*

(a) $\cos(195°)$

(b) $\sin(-15°)$

(c) $\sin\left(\dfrac{5\pi}{12}\right)$

3. Simplify the expression:

$$\frac{\cos(x+y)}{\sin x \cos y} =$$

4. Use the sum formula for sine to write $\sin(2\theta)$ in terms of $\sin\theta$ and $\cos\theta$.

Theorem: Double-Angle Formulas

- $\sin(2\theta) = 2\sin\theta\cos\theta$
- $\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$
- $\cos(2\theta) = \cos^2\theta - \sin^2\theta = 1 - 2\sin^2\theta = 2\cos^2\theta - 1$

5. Suppose that θ lies in Quadrant II, and $\sin\theta = \frac{1}{4}$. Determine each of $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$.

6. Use the sum formula for cosine and the main Pythagorean identity to prove that $\cos(2\theta) = 1 - 2\sin^2\theta$.

7. Prove that $\sin^2\theta = \frac{1}{2}(1-\cos(2\theta))$ and $\cos^2\theta = \frac{1}{2}(1+\cos(2\theta))$

Theorem: Half-Angle Formulas

- $\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$
- $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

The choice of $+$ or $-$ is determined by the quadrant of the angle $\frac{\theta}{2}$.

8. Find the *exact* value of each of the following. *Do not give a decimal approximation.*

(a) $\cos(22.5^\circ)$

(b) $\sin(-\dfrac{\pi}{8})$