Calculus 1 — Outline for the Final Exam

Main ideas

Old

A. Limits (one-sided, two-sided, and at infinity)

B. Continuity

C. Definition of the derivative of a function and tangent lines

D. Derivative rules including product, quotient and chain rules

E. Derivative formulas for power, trigonometric, inverse trig., exponential, and logarithmic functions

F. Implicit differentiation

G. Interpreting first and second derivatives and using them to sketch curves

H. Applications of differentiation to *related rates* and *optimization*

I. L'Hôpital's rule

New

J. Definition of the definite integral and the (net) area under a curve

K. Fundamental Theorem of Calculus

L. Net change of a function

M. Indefinite integrals and antiderivatives

N. u-substitution

Skills you should have

1. Be able to compute basic limits graphically, algebraically, and from a table of numbers

2. Be able to state the definition of a function being continuous at a, which is that (1) $f(a)$ exists, (2) $\lim_{x\to a} f(x)$ exists, and (3) $\lim_{x\to a} f(x) = f(a)$

3. Be able to compute derivatives and tangent lines using the definition of the derivative; that is, using $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ instead of the derivative rules

4. Be able to determine graphically of a function is continuous or differentiable

5. Be able to compute derivatives and tangent lines using the various derivative rules and formulas

6. Be able to compute derivatives of implicityly defined functions, e.g. $\sin(xy) = x^2 + e^y$

7. Be able to interpret the first and second derivatives of a function f

- **(a)** Connection between f' positive/negative and f increasing/decreasing
- **(b)** Connection between f'' positive/negative and f concave up/down

(c) Finding local extrema and inflection points

(d) Use first and second derivatives (and asymptotes, limits at infinity, ...) to sketch graphs

8. Be able to solve related rates problems

(a) Do not confuse these problems with optimization problems!

(b) Know what is constant with respect to time and what is not

9. Be able to optimize a function f on an interval I, i.e. find absolute maximums and minimums

(a) Finding and testing critical points of f and endpoints of I (and what to do if I is not closed)

(b) Be able to work in the context of a **word problem** where YOU have to determine the function to optimize and the interval to optimize over

10. Using L'Hôpital's rule

(a) The rule only applies to limits of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$

(b) Know how to deal with limits of the form $0 \cdot \infty$ by "flipping something over"

(c) Know how to deal with limits of the form 0^∞, 1^∞, 0^0, and ∞^0 using logarithms

11. Be able to express $\int_a^b f(x)\,dx$ (or the net area under a function) as a limit of Riemann sums

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

by filling in for Δx, x_i, and f.

12. Be able to approximate $\int_a^b f(x)\,dx$ (or the net area under a function) using R_n, L_n, or M_n for a fixed value of n (like $n = 6$).

13. Be able to evaluate a definite integral $\int_a^b f(x)\,dx$ using...

(a) geometry: thinking of $\int_a^b f(x)\,dx$ as the net area between f and the x-axis from a to b

(b) FTC 2: $\int_a^b f(x)\,dx = F(b) - F(a)$ for F an antiderivative of f

14. Be able to state FTC 1 and be able to work with "area functions".

15. Be able to compute the net change of a quantity Q using FTC 2, i.e. $Q(b) - Q(a) = \int_a^b \frac{dQ}{dt}\,dt$.

16. Be able to find indefinite integrals and antiderivatives

(a) know common antiderivatives

(b) practice u-substitution

(c) don't forget the "$+ C$"

How to study

I. Review core topics

II. Work *lots* of problems all of the way through—focus on WebAssign problems and Group Work problems

III. Practice doing several problems in a short amount of time (by timing yourself)

IV. Come talk with me if you have any questions