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Author 2 $\qquad$
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## Definition: Derivatives of the Inverse Trigonometric Functions

- $\sinh x=\frac{1}{2}\left(e^{x}-e^{-x}\right)$
- $\cosh x=\frac{1}{2}\left(e^{x}+e^{-x}\right)$
- $\tanh x=\frac{\sinh x}{\cosh x}$
- $\operatorname{coth} x=\frac{\cosh x}{\sinh x}$
- $\operatorname{sech} x=\frac{1}{\cosh x}$
- $\operatorname{csch} x=\frac{1}{\sinh x}$


## Theorem: Derivatives of the Inverse Trigonometric Functions

- $(\sinh x)^{\prime}=\cosh x$
- $(\cosh x)^{\prime}=\sinh x$
- $(\tanh x)^{\prime}=\operatorname{sech}^{2} x$
- $(\operatorname{coth} x)^{\prime}=-\operatorname{csch}^{2} x$
- $(\operatorname{sech} x)^{\prime}=-\operatorname{sech} x \tanh x$
- $(\operatorname{csch} x)^{\prime}=-\operatorname{csch} x \operatorname{coth} x$

1. Use the definitions of $\sinh x$ and $\cosh x$ to show that $\cosh ^{2} x-\sinh ^{2} x=1$.
2. Find the derivative of $f(x)=\sinh \left(x^{2}\right)(7 x-\ln (x))$.
3. A telephone line hangs between two poles $14 m$ apart in the shape of a catenary $y=20 \cosh \left(\frac{x}{20}\right)-15$, where $x$ and $y$ are measured in meters. Find the slope of the curve where it meets the left pole.


Definition: Linear Approximation
The linear approximation of $f(x)$ at $x=a$ is

$$
f(x) \approx f(a)+f^{\prime}(x)(x-a)
$$

4. Use a linear approximation of $\sqrt{x}$ at an appropriate value $a$ to estimate $\sqrt{100.5}$.
