AUTHOR 1 _____

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# $22 -$ Increasing/Decreasing & Concavity

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> **Definition: Increasing/Decreasing & Concavity**
>
> Let $f$ be a function and $I$ an interval.
>
> - $f$ is **increasing** on $I$ if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$.
>
> - $f$ is **decreasing** on $I$ if if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$.
>
> - $f$ is **concave up** on $I$ if the graph of $f$ lies *above* all of its tangent lines on $I$.
>
> - $f$ is **concave down** on $I$ if the graph of $f$ lies *below* all of its tangent lines on $I$.
>
> - An **inflection point** of $f$ is a point where $f$ is continuous and the concavity of $f$ changes.

**1.** The graph of $f(x)$ is below.
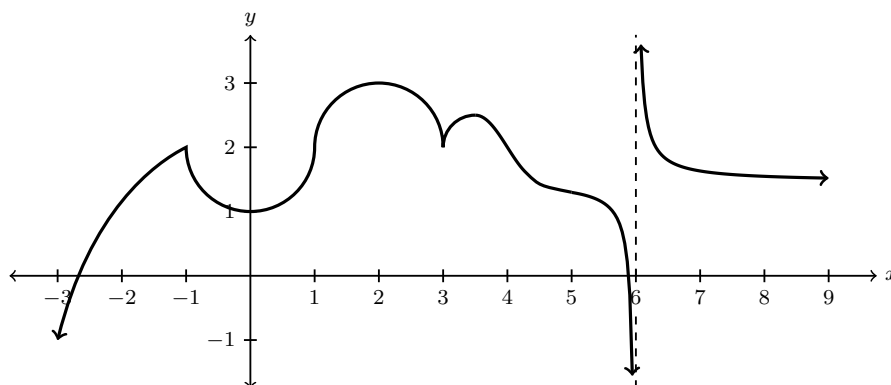


**(a)** On what intervals is $f$ increasing?

**(b)** On what intervals is $f$ decreasing?

**(c)** List the $x$-values of the local extrema?

**(d)** On what intervals is $f$ concave up?

**(e)** On what intervals is $f$ concave down?

**(f)** List the $x$-values of the inflection points?

**2.** On the graph above, draw four tangent lines anywhere between $x = 1$ and $x = 3$. Describe how the slopes of the tangent lines are changing as $x$ varies from 1 to 3. What does this mean about $f'(x)$?

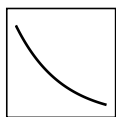**Theorem: Determining Intervals of Increasing/Decreasing & Concavity**

Let $f$ be a function and $I$ an interval.

- $f$ is increasing on $I$ if $f'(x) > 0$ on $I$.

- $f$ is decreasing on $I$ if $f'(x) < 0$ on $I$.

- $f$ is concave up on $I$ if $f''(x) > 0$ on $I$.
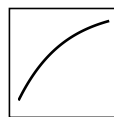
- $f$ is concave down on $I$ if $f''(x) < 0$ on $I$.

**3.** For each blank below, choose the shape of the graph described by the conditions on $f'$ and $f''$.

| I | II | III | IV |
|---|----|-----|----|

$f'(x) < 0$
$f''(x) > 0$ _____
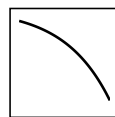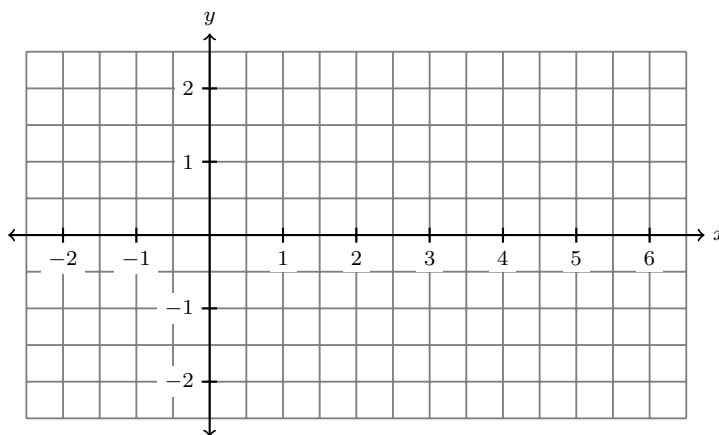
$f'(x) > 0$
$f''(x) < 0$ _____

$f'(x) > 0$
$f''(x) > 0$ _____

$f'(x) < 0$
$f''(x) < 0$ _____

**4.** Draw the graph of a function $f$ that has the given properties:

- $f$ is discontinuous at $x = 4$

- $f'(x) < 0$ **only** when $3 < x < 4$,

- $f''(x) > 0$ **only** when $0 < x < 2$

- $\lim\limits_{x \to \infty} f(x) = 1$

**(a)** List the $x$-values of the local extrema?        **(b)** List the $x$-values of the inflection points?