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## 23 - Finding Extrema \& Inflection Points

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## Strategy: Finding Increasing/Decreasing, Concavity, Extrema, \& Inflection Points

Suppose you want to determine when $f$ is increasing/decreasing, where $f$ concave up/down, what are the local extrema, or what are the inflection points.

## I. Increasing/Decreasing \& Local Extrema.

1. Compute $f^{\prime}(x)$.
2. Find all $x$-values where $f^{\prime}(x)=0$ or $f^{\prime}(x)$ DNE, and put them on a number line. Here's an example if $f^{\prime}(1)=0, f^{\prime}(3)$ DNE, and $f^{\prime}(4)=0$.

3. Choose $x$-values on either side of the values you found in the previous step. Compute $f^{\prime}$ at each of these values to determine if $f^{\prime}$ is positive $(+)$ or negative $(-)$ on each interval.


$$
\begin{array}{ccc}
f^{\prime}\left(b_{1}\right)>0 & - \\
f^{\prime}\left(b_{2}\right)>0 & + \\
f^{\prime}\left(b_{3}\right)<0 & - \\
f^{\prime}\left(b_{4}\right)>0 & -
\end{array}
$$

4. Increasing/Decreasing: $f$ is increasing if $f^{\prime}$ is $+; f$ is decreasing if $f^{\prime}$ is - .
5. Local Extrema: local max at $x=a$ if $f^{\prime}$ changes from + to - (and $a$ is in domain of $f$ ); local min if $f^{\prime}$ changes from - to + at $a$ (and $a$ is in domain of $f$ ).

## II. Concave Up/Down \& Inflection Points.

1. Compute $f^{\prime \prime}(x)$.
2. Find all $x$-values where $f^{\prime \prime}(x)=0$ or $f^{\prime \prime}(x)$ DNE, and put them on a number line.

3. Choose $x$-values on either side of the values you found in the previous step. Compute $f^{\prime \prime}$ at each of these values to determine if $f^{\prime \prime}$ is positive $(+)$ or negative $(-)$ on each interval.

4. Concave Up/Down: $f$ is concave up if $f^{\prime \prime}$ is $+; f$ is concave down if $f^{\prime \prime}$ is - .
5. Inflection Points: inflection point at $x=a$ if $f^{\prime \prime}$ changes sign (and $a$ is in domain of $f$ ).
6. Let $f(x)=x^{4} e^{-x}$. Find all intervals of increasing or decreasing, all intervals where the graph is concave up or concave down, all local extrema, and all inflections points. (Your answers go below.) Also, use this information to sketch the graph of $y=f(x)$.

- Intervals where $f$ is increasing
- Intervals where $f$ is decreasing
- Local extrema
- Intervals where $f$ is concave up
- Intervals where $f$ is concave down
- Inflection points


