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24 – L'Hôpital's Rule

Theorem: L'Hôpital's Rule (HR)

Suppose f and g are differentiable and $g'(x) \neq 0$ near a, except possibly at a. If $\lim_{x\to a} \dfrac{f(x)}{g(x)}$ has the form $\frac{\infty}{\infty}$ or $\frac{0}{0}$, then

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \lim_{x\to a} \frac{f'(x)}{g'(x)}$$

provided the limit on the right exists or is $\pm\infty$. Moreover, HR remains true with $x \to a$ replaced by any of $x \to a^+$, $x \to a^-$, $x \to \infty$, or $x \to -\infty$.

1. Find the following limits.

(a) $\displaystyle\lim_{x\to 1} \frac{x\sin(x-1)}{2x^2 - x - 1}$

(b) $\displaystyle\lim_{x\to -\infty} x^2 e^x$

(c) $\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right)$

2. Consider the limit $\lim_{x \to 0^+} (\cos x)^{1/x^2}$.

(a) What does "direct substitution" yield for the limit $\lim_{x \to 0^+} (\cos x)^{1/x^2}$?

(b) Find $\lim_{x \to 0^+} (\cos x)^{1/x^2}$ (by using logarithms).