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# 03 – Introduction to Limits

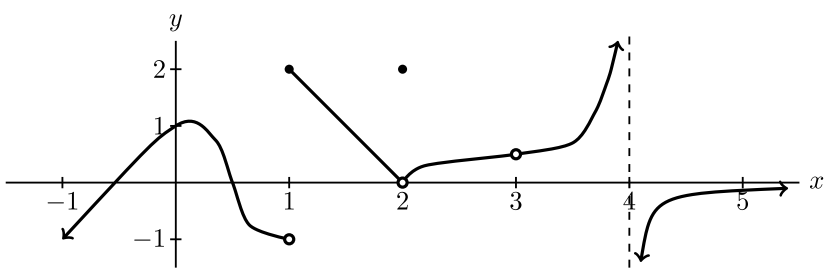
**1.** Suppose $f(x)$ is a mystery function for which some values are given (approximately) below.

| $x$ | $-2$ | $-1$ | $-0.5$ | $-0.1$ | $0$ | $0.1$ | $0.5$ | $1$ | $2$ |
|---|---|---|---|---|---|---|---|---|---|
| $f(x)$ | $0.45$ | $0.84$ | $0.96$ | $0.998$ | | $0.998$ | $0.96$ | $0.84$ | $0.45$ |

**(a)** Based on this data, what do you think is the value of $f(0)$?

**(b)** Look in the lower-left corner of page 2 to see what the mystery function is. Does this change your answer about $f(0)$? Explain.

**2.** Suppose the graph of a function $h(x)$ is given below. Find the value of each of the following below.




**(a)** $h(0) =$

**(b)** $h(1) =$

**(c)** $h(2) =$

**(d)** $h(3) =$

**(e)** $h(4) =$

So, there can be a **big** difference between the *actual value* of a function at a number $a$ and what the *values are approaching* near $a$. The next definition captures what the values are approaching.

**Definition: Limits (Informally)**

We write $\lim_{x\to a} f(x) = L$ if the value of $f(x)$ can be made to stay arbitrarily close to $L$ for all $x$ sufficiently close to $a$, *but not equal to* $a$.

- $\lim_{x\to a} f(x)$ is asking where the outputs of the function *appear* to be going as $x$ approaches $a$.
- $\lim_{x\to a} f(x) = L$ is read "the limit of $f(x)$ as $x$ approaches $a$ is $L$".

**3.** Answer the following questions about the functions $f$ and $h$ from above.

**(a)** $\lim_{x\to 0} f(x) =$

**(b)** $\lim_{x\to 0} h(x) =$

**(c)** $\lim_{x\to 1} h(x) =$

**(d)** $\lim_{x\to 2} h(x) =$

**(e)** $\lim_{x\to 3} h(x) =$

**(f)** $\lim_{x\to 4} h(x) =$

## Definition: One-Sided Limits (Informally)

**From the left:** only considering $x$-values **less than** $a$, we define $\lim_{x \to a^-} f(x) = L$.

**From the right:** only considering $x$-values **greater than** $a$, we define $\lim_{x \to a^+} f(x) = L$.

**4.** Answer the following questions about the functions $f$ and $h$ (from the previous page).

**(a)** $\lim_{x \to 0^-} f(x) =$

**(b)** $\lim_{x \to 1^+} h(x) =$

**(c)** $\lim_{x \to 1^-} h(x) =$

**(d)** $\lim_{x \to 2^+} h(x) =$

**(e)** $\lim_{x \to 2^-} h(x) =$

**(f)** $\lim_{x \to 4^-} h(x) =$

**5.** Investigate $\lim_{x \to 0^+} \sin\left(\frac{\pi}{2x}\right)$ by following the steps below.

**(a)** Fill in the table below, and use it to make a guess about $\lim_{x \to 0^+} \sin\left(\frac{\pi}{2x}\right)$. (I did the first one.)

| $x$ | $\frac{1}{10}$ | $\frac{1}{100}$ | $\frac{1}{1000}$ | 0 |
|---|---|---|---|---|
| $\sin\left(\frac{\pi}{2x}\right)$ | $\sin\left(\frac{\pi}{\frac{1}{5}}\right) = \sin(5\pi) = \boxed{0}$ | | | |

Use the table to make a guess about $\lim_{x \to 0^+} \sin\left(\frac{\pi}{2x}\right) =$

**(b)** Find the value of $\sin\left(\frac{\pi}{2x}\right)$ when $x = \frac{1}{1001}$. Does this change your guess about $\lim_{x \to 0^+} \sin\left(\frac{\pi}{2x}\right)$?

**(c)** Use your phone to graph $\sin\left(\frac{\pi}{2x}\right)$ at `www.desmos.com` or `www.wolframalpha.com`. Give your final answer to $\lim_{x \to 0^+} \sin\left(\frac{\pi}{2x}\right)$ below. Make sure to explain!

**6.** Find the following given that $g(x) = \begin{cases} \ln x, & \text{if } 0 < x < 1 \\ e^{x-1} - 1, & \text{if } 1 < x \leq 2. \\ x + e, & \text{if } x > 2 \end{cases}$

**(a)** $\lim_{x \to 1^+} g(x) =$

**(b)** $\lim_{x \to 1^-} g(x) =$

**(c)** $\lim_{x \to 1} g(x) =$

**(d)** $\lim_{x \to 2^+} g(x) =$

**(e)** $\lim_{x \to 2^-} g(x) =$

**(f)** $\lim_{x \to 2} g(x) =$