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18 – Hyperbolic Trig & Linear Approx.

Definition: Derivatives of the Inverse Trigonometric Functions

- $\sinh x = \dfrac{1}{2}(e^x - e^{-x})$
- $\cosh x = \dfrac{1}{2}(e^x + e^{-x})$
- $\tanh x = \dfrac{\sinh x}{\cosh x}$
- $\coth x = \dfrac{\cosh x}{\sinh x}$
- $\operatorname{sech} x = \dfrac{1}{\cosh x}$
- $\operatorname{csch} x = \dfrac{1}{\sinh x}$

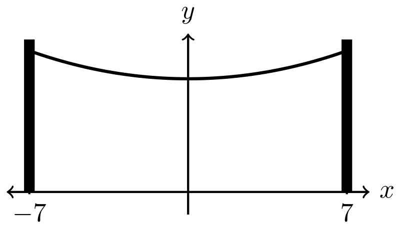
Theorem: Derivatives of the Inverse Trigonometric Functions

- $(\sinh x)' = \cosh x$
- $(\cosh x)' = \sinh x$
- $(\tanh x)' = \operatorname{sech}^2 x$
- $(\coth x)' = -\operatorname{csch}^2 x$
- $(\operatorname{sech} x)' = -\operatorname{sech} x \tanh x$
- $(\operatorname{csch} x)' = -\operatorname{csch} x \coth x$

1. Use the definitions of $\sinh x$ and $\cosh x$ to show that $\cosh^2 x - \sinh^2 x = 1$.

2. Find the derivative of $f(x) = \sinh(x^2)(7x - \ln(x))$.

3. A telephone line hangs between two poles $14m$ apart in the shape of a catenary $y = 20\cosh\left(\frac{x}{20}\right) - 15$, where x and y are measured in meters. Find the slope of the curve where it meets the left pole.



Definition: Linear Approximation

The **linear approximation** of $f(x)$ at a is

$$f(x) \approx f(a) + f'(x)(x - a) \quad \text{for } x \text{ near } a.$$

4. One way to estimate $\sqrt{100.5}$ is to say $\sqrt{100.5} \approx \sqrt{100} = 10$. Instead, use the linear approximation of $\sqrt{x}$ at 100 to estimate $\sqrt{100.5}$. Is your estimate using the linear approximation more or less accurate than the simple approximation $\sqrt{100.5} \approx 10$?