Author 1 ______________________

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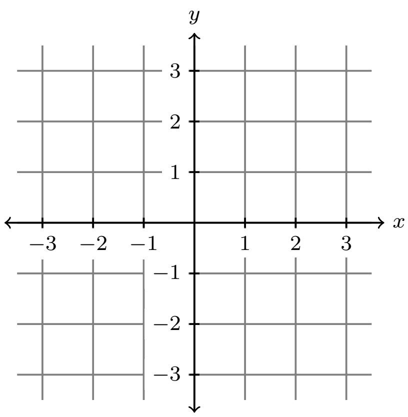
27 – Definite Integral

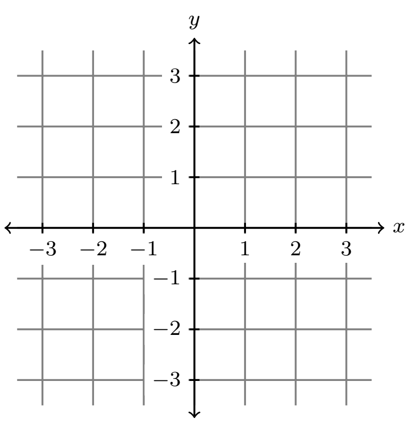
Theorem: Evaluating Definite Integrals Geometrically

$$\int_a^b f(x)\,dx = \begin{pmatrix}\text{total area under } f \text{ and} \\ \textit{above} \text{ the } x\text{-axis}\end{pmatrix} - \begin{pmatrix}\text{total area above } f \text{ and} \\ \textit{below} \text{ the } x\text{-axis}\end{pmatrix}$$

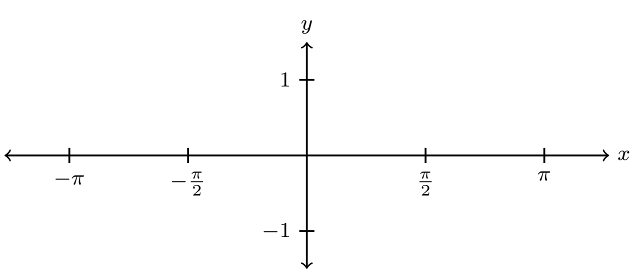
1. Graph $f(x) = x - 1$ over $[-1, 2]$, and evaluate $\int_{-1}^{2} (x-1)\,dx$ by interpreting it as (net) area.



2. Graph $f(x) = \sqrt{4 - x^2}$ over $[-2, 2]$, and evaluate $\int_{-2}^{2} \sqrt{4 - x^2}\,dx$ by interpreting it as (net) area.



3. Graph $f(x) = \sin x$ over $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin x\,dx$ by interpreting it as (net) area.



Theorem: Evaluating Definite Integrals Algebraically

If f is integrable, then

$$\int_a^b f(x)\,dx = \lim_{n\to\infty}\left(\sum_{i=1}^{n} f(x_i)\Delta x\right)$$

where $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\left(\frac{b-a}{n}\right)$.

4. Consider the integral $\displaystyle\int_1^3 \frac{1}{1+x^2}\,dx$.

(a) Estimate the integral using R_4 (4 subintervals with right-hand endpoints as sample points).

(b) Express the integral as a limit of Riemann sums. (But, do not compute it.)

Theorem: Computing Displacement from Velocity

If $v(t)$ gives the velocity of of an object at time t, then the *(net) displacement*, D, of the object from $t = a$ to $t = b$ is

$$D = \int_a^b v(t)\,dt.$$

5. Suppose that the velocity of a space shuttle t seconds after takeoff is modeled by $v(t) = 0.125t^2 - 4.8t$, in m/s. This model is only valid in the first 124 seconds while the rocket boosters are assisting.

(a) What is the velocity of the shuttle after 124 seconds?

(b) Write down (but don't compute) a definite integral that expresses the distance traveled by the rocket in the first 124 seconds.

(c) Estimate the distance traveled by the rocket in the first 124 seconds using R_4 (and a calculator).